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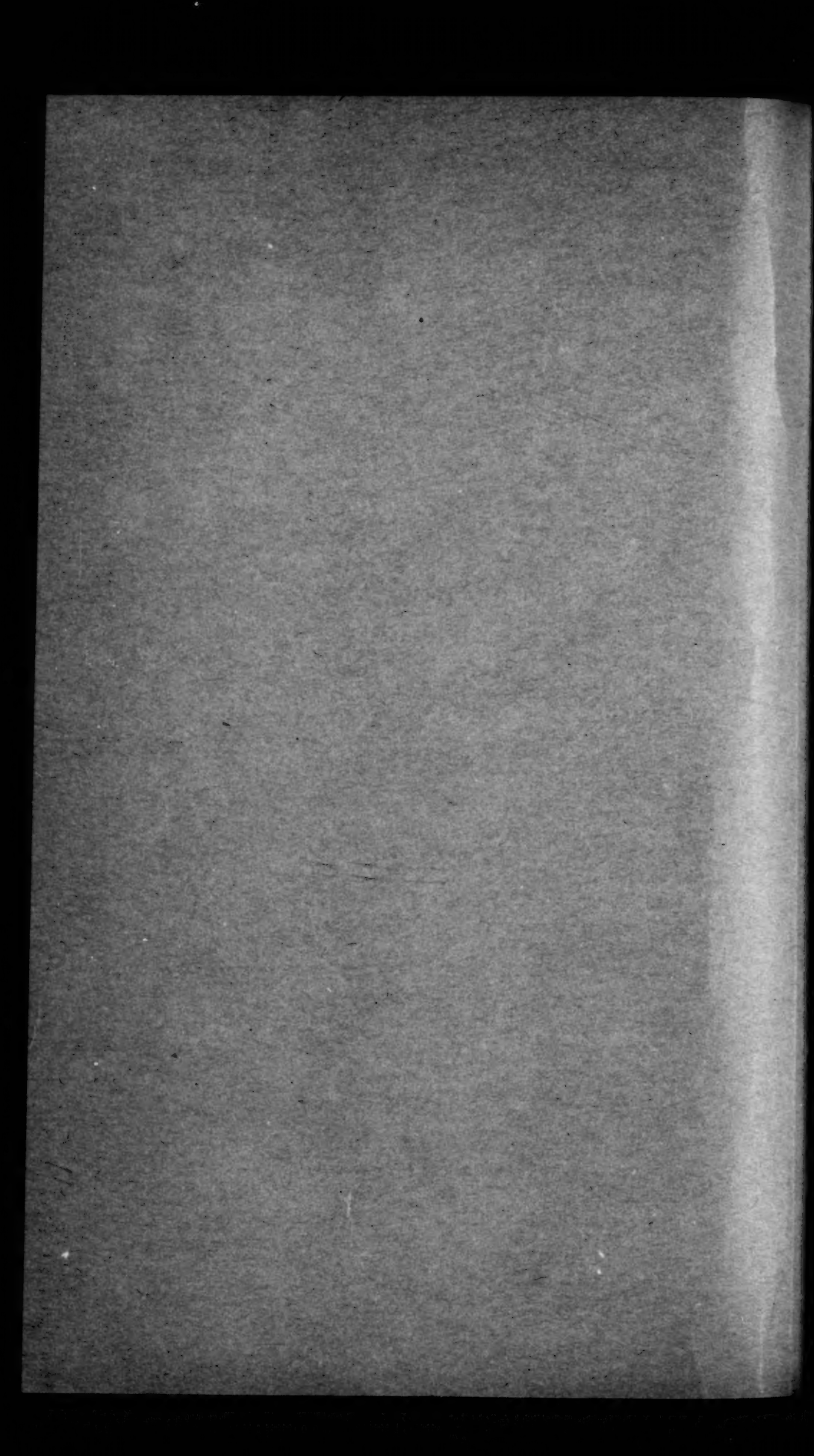

NEW SERIES, NO. 174 (VOL. XXVI) JUNE, 1931

JOURNAL OF THE AMERICAN STATISTICAL ASSOCIATION

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A FORECASTING INDEX FOR BUSINESS[1]

By Bradford B. Smith

We are probably all in agreement that when the volume of buying of goods and services in the country is relatively large the nation enjoys prosperity. And similarly a sharp shrinkage in purchasing transforms prosperity into depression. The magnitude of the money flow from buyer to seller measures the degree of our prosperity or depression, for after all a large volume of buying of goods is, *per se*, a large volume of business, and vice versa. It is but a short step from this concept to a visualization of the other major flow of money in our economic body, the flow of money from industry to individuals. In the aggregate, industry disburses huge sums of money to the individuals of the nation in the form of wages, salaries, dividends, interest, rent and the like. It is these sums which enable the individuals to turn about and in their capacity as consumers to purchase the goods which industry offers for sale; and in payment, the money is returned to industry. Thus we can think of the nation's business as a vast mechanism in endless motion with these two great flows of money, joined into a continuous circuit, acting as the motivating force. There is industry disbursing money as wages and the like in payment for services rendered in the production of goods, and the individuals returning this money to industry in payment for goods purchased.

As the blood of the body may transmit disease from one part to another, so the money circulation in the economic body transmits and augments influences brought to bear on it. Thus suppose that money on its way to purchase automobiles is diverted and not used for purchasing goods. The sales of the auto manufacturers are reduced. Their reduced sales cause them to release employees. The employees are

[1] Paper delivered at the Ninety-Second Annual Meeting of the American Statistical Association at Cleveland, Ohio, December 31, 1930.

then less able to purchase shoes, clothes, tires and the like. This further restricts purchasing power, prices of goods are reduced in an effort to sell them in the shrinking markets and so the nation's feet are set on the vicious spiral leading down into business depression. On the other hand, the injection of new money into the circuit increases purchases of goods, the increased demand leads to increased employment and larger wage disbursements; this further amplifies purchasing power and so the vicious downward spiral is replaced by the delicious upward spiral. If the injection of new money is more rapid than goods can be produced, price inflation results. The money circuit in our business mechanism is capable of transmitting with self-reinforcing vigor the influence of additions to or withdrawals from the circuit, of relatively small sums. I am inclined to believe that the direct if not the ultimate, the practical if not the philosophical, cause of our business cycles is inherent in the ability of our banking system to make these initial additions to and withdrawals from the money flows in our economic body. It is through measuring of these money flows that the author has constructed a forecasting index for business, and through the accuracy of the index attempted to establish the validity of this thesis. Let us first briefly consider the mechanics of these money additions and withdrawals.

A contractor goes to a bank to make a loan. The bank does not ordinarily give him the hard cash. It takes his note, and a lien on his property, and then credits his deposit account in the amount of the loan. What happens is that bank deposits increase, and these deposits are just as effective for purchasing goods as though they had been created by the bringing in and deposit of actual money. There is a net addition to the purchasing power of the people. More strictly speaking, the asset values which the contractor put up with the bank as security for his loan have been temporarily transformed into "coin of the realm" and are thus effective in enhancing purchasing power. The contractor's purchasing power temporarily exceeds his income by the amount of his loan. Or, taking another illustration, the bank may credit the deposit account of a railroad in return for bonds which the railroad issues. And this new money is soon sent scurrying along the money channels, increasing demand, breeding prosperity, as the railroad pays it out in the process of construction and the like. One of these methods is loans, the other is investments and these are the two pipe lines by means of which the banking system makes additions to or withdrawals from the money circuit, thus initiating upward and downward spirals giving prosperity or depression.

As the expanding of loans and investments, that is bank credit, pro-

ceeds there comes a time, of course, when the process needs must be slowed down. This is because the expanding of credit increases deposits and the laws of our land dealing with bank reserves provide that deposits shall not be more than about ten times the amount of reserves which the banks have. As this limit is approached the banks are restrained in the freedom with which they make loans or investments. When this occurs, those most in need of accommodation bid up the interest rate in an effort to secure unto themselves such accommodation as is available and thus a period of higher interest rates comes into being. It should be mentioned at this point that a period of high interest rates may also be created by reducing bank reserves, as well as by expanding bank credit. The ten-to-one rule becomes applicable in either instance. Although the effects of these high interest rates vary somewhat according to the method by which they are brought about, it is through examining these effects that we gain the greatest insight into the workings of the business cycle for forecasting purposes.

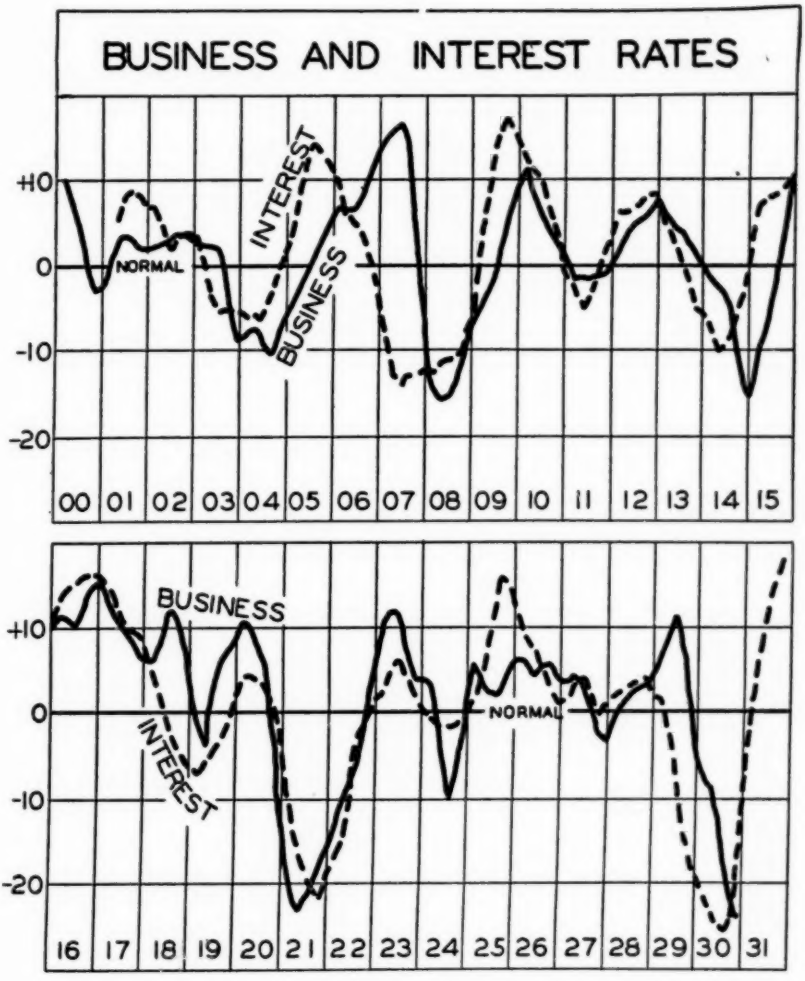
For our purposes we may distinguish two kinds of borrowing, already briefly mentioned. The first is represented by long-term bonds held as investments by banks. The second is the short-time commercial or collateral loan. The first is highly affected by interest rates; the second is relatively little influenced by interest rates but is highly affected by credit availability. That is, industry is loath to tie itself up to heavy fixed charges over a long period of time as it would have to do if it floated bonds in periods of high interest rates; for short periods it will pay high rates if it can only get the money. The writer is inclined to believe that it is the influence of interest cost on long-term borrowing which largely regulates our minor business cycles, and the influence of credit availability on short-time borrowing which largely directs the course of our major business cycles. Considering the minor business cycle first, as interest rates are bid up to higher levels by short-time borrowers, the flotation of long-term bonds is restricted. Money secured from these flotations is characteristically used for the construction of factories, railways, roads, houses and other forms of building. As the flotations are curtailed the sums of money being disbursed to the people engaged in this construction work are correspondingly restricted. The effect becomes increasingly important as the projects in process are completed and their place not taken by new ones. The reduced purchasing of this group of consumers lessens the demand for industry's products and industrial production is correspondingly curtailed. Further unemployment appears and the vicious downward spiral is initiated. But as business activity slows down, the demand for bank accommodation for the financing of current production on short-term

loans is reduced. Bank credit contracts as loans are allowed to run off, and the banks again find themselves able to offer funds at low interest rates to those who would float bonds. The revival of these flotations starts up construction again, thus increasing purchasing power of the people in that industry. Furthermore—and this is important—this increased purchase and consumption of goods is accomplished without a commensurate increase in production of goods offered for immediate sale, and thus constitutes a potent force for cleaning up surplus stocks of these current goods. The stimulus to industry is prompt and effective and thus depression gives way to recovery. Bank credit again expands until once more the legal limits are approached, interest rates get high enough to suppress bond flotations and then the whole thing is repeated. This is, in my opinion, the way the minor business cycles work out, with their characteristic gentle fluctuations between moderate prosperity and mild depression.

Occasionally great speculative opportunities in conjunction with abundant credit supplies expand these moderate fluctuations into the great business boom and its inevitable sequel, the major business depression. In periods of speculative fervor, credit is expanded, interest rates become high, and long-time bond financing is deferred. But the reduced dissemination of purchasing power through the construction industry is more than offset by the extension of purchasing power through the speculative markets, the expanding credit being disseminated in the form of speculative profits. The expansion goes on in the face of high interest rates, people paying high charges on short-time collateral loans in the hope of securing great speculative profits. This proceeds until the absolute legal credit limitations are approached and banks refuse to renew or to make further loans. The interest rate sometimes shoots to unbelievable heights. We then have not only costly credit but scarcity of credit. There comes a rush to sell the speculatively held securities in order to pay off the straining bank loans falling due. The consumers' closest contact with this rush, which soon becomes compulsory, is through the perhaps too familiar margin call of the broker. There is a tremendous shrinkage of credit as loans are paid down; and we have seen this means a tremendous shrinkage in purchasing power. If we individually have to pay much money to brokers on margin calls, our ability to purchase automobiles is correspondingly impaired. Despite the prompt easing of interest rates, as the credit volume contracts, the reviving effect of new construction, which is so important in cleaning up surplus stocks of current goods and stimulating a recovery in business, is slow. For the long period of high interest rates greatly shrinks the volume of construction projects and

they cannot be started up again as swiftly as boom prosperity can be transformed into depression. During the interval the nation experiences dismal depression, as we now are doing.

CHART I



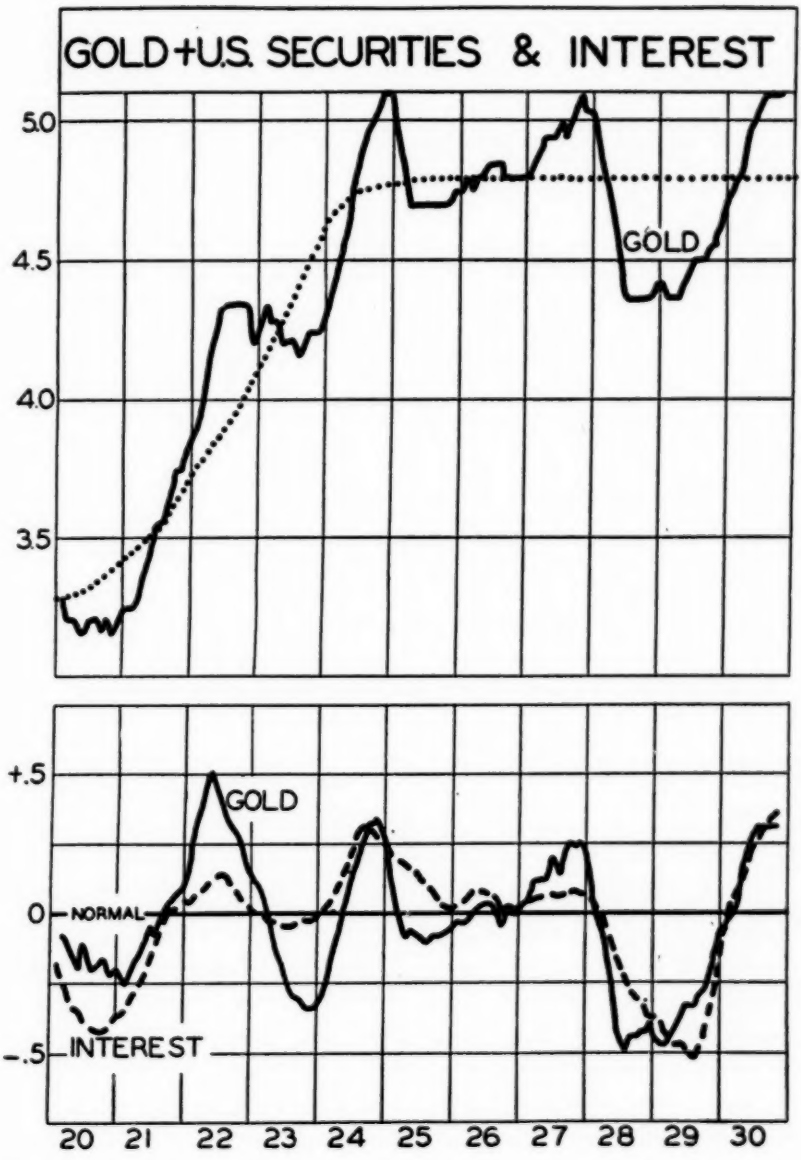
If all this is true we should expect interest rates to show some forecasting significance, and so they do. Chart I has been constructed for the purpose of showing this relationship. In this diagram there are two parts, an upper and a lower. The upper part deals with the years from 1900 through 1915 and the lower part carries forward the picture from

1916 to the present time. The irregular solid line represents the familiar course of business activity month by month throughout the entire thirty-year span. For the first twenty years the index is that of the American Telephone and Telegraph Company. For the later years it is an index prepared by the Cleveland Trust Company. The curve has been smoothed slightly by a five-month centered average. The dashed line in the diagram represents the course of interest rates for this period.[1] This line is an average of commercial paper and time loan rates. Purely seasonal variations prior to 1914 have been removed. In the preparation of this series the yields of high-grade long-term bonds have been taken as a normal, so that the curve really represents the deviations between short-time interest rates and long-term bond yields. These deviations have also been slightly smoothed by a five-month average. Finally the deviations have been charted upside down. When the curve is above the zero level it signifies that interest rates were low, not high; and when below the zero level, interest rates were high. There are several instances where the agreement between the two curves leaves much to be desired, but on the whole, and considering the long period of years, there does seem to be a certain amount of similarity between the fluctuations of the two lines. Now the interesting thing is that the interest rate line has been bodily moved forward one year, so that the data plotted in the 1921 space on the diagram, for example, are really data for 1920. In other words, this interest rate line seems to be a one-year forecaster of business activity. It seems to be a factor which is well worth incorporating in a forecasting index for business.

At this point we should recall that high interest rates may be brought about by shrinkage in bank reserves as well as by expansion of credit, and vice versa. Furthermore, it is not entirely logical to suppose that the consequences of high interest rates brought about by the shrinkage of reserves would be identical with those brought about by great credit expansion involving price inflation and a large degree of indebtedness subsequently to be paid down. For that reason it seems advisable to examine the relationships between fluctuations in bank reserves and interest rates, with a view of introducing a compensation element or factor in the interest rate series. For reasons which time does not permit going into, but which can be fairly definitely substantiated, the most important element influencing interest rates through bank reserves may be taken as the sum of the monetary gold in the country

[1] The violent fluctuations following the declaration of war in 1914 have been bridged over for a six month period, as was also necessary in the case of security prices, mentioned later, owing to the closing of the New York Stock Exchange.

CHART II

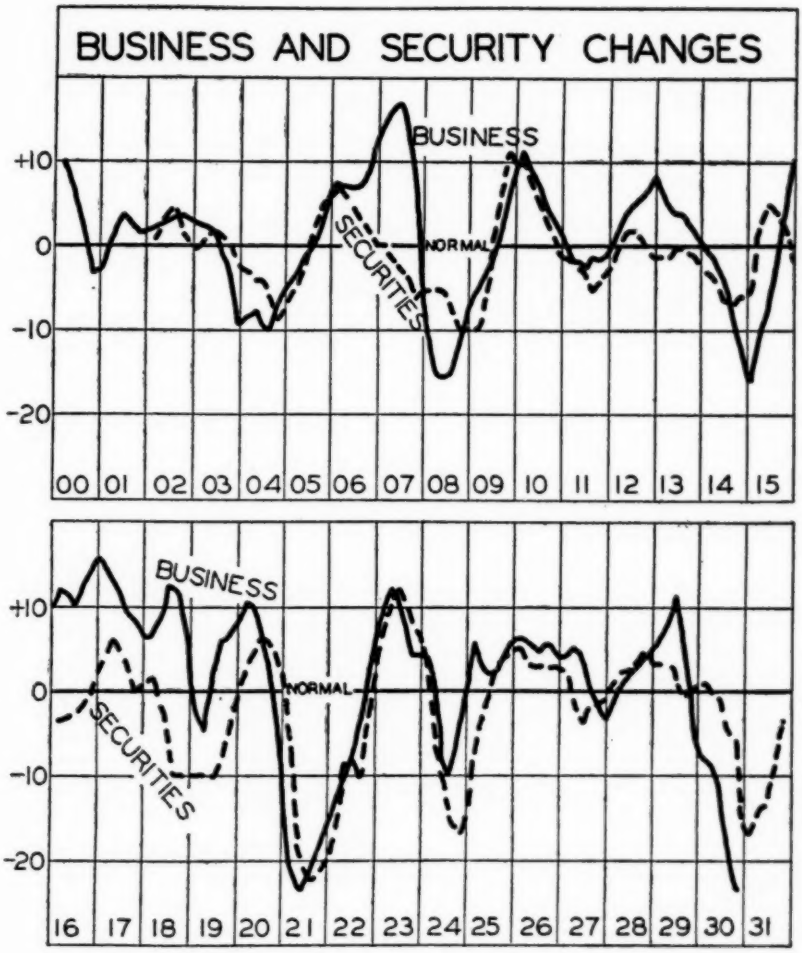


added to the Federal Reserve Bank holdings of United States securities. In Chart II, as in Chart I, there is an upper and a lower part. In the upper part the irregular solid line represents month by month since 1920 the fluctuation in this sum of gold and securities. The smoother

line represents the trend. Its most noteworthy aspect is the sharpness with which it rises in the earlier years during the time when the United States was becoming the world's chief holder of monetary gold. For the last few years the trend lines move horizontally. In the lower part of the diagram the deviations of the gold and securities data from the trend line are represented by the solid line for the same period of years. The dashed line is the interest rate curve just as it was charted on the previous diagram except that here there is no lead or lag. The two curves seem to have a definite family resemblance and thus substantiate the suggestion that interest rates have been caused to move by more than the fluctuations in bank credit alone. So this gives a second factor to be included in the index for forecasting business, introduced as a corrective element for the interest rate data.

We recall now that one of the most effective methods of expanding bank credit, and thus purchasing power, is through rising security markets. This applies to both bond markets and stock markets. There are several reasons why a rising bond market portends better business, other things being equal. In the first place a rising bond market is a good time in which to float bonds, and bond flotations, it has been seen, are the preliminary to construction work which is so important in lifting business out of depression. In the second place the fact that a bond market is rising means that holders of bonds have profits accruing to them. In the case of individuals this makes them better spenders of money; in the case of banks it makes them more liberal in their extension of credit and in the financing of new ventures. The fact that the stock market is rising means not only that there is more optimism and hence less restraint in spending, but also a rising stock market is perhaps the most effective means of quickly expanding bank credit and widely distributing effective purchasing power in the form of speculative profits. Rising markets are generally financed by expanding credit and speculative profits are easily spent thus providing stimulus to business. To reduce this concept to statistical terms a measurement is needed of the rapidity with which security markets rise or fall. The measurement used in this case was the number of points which the securities would rise during a period of one year if they continued to rise at the same rate which the trend over the past year would indicate. Technically speaking, the measurement was the inverted twelve-months' trailing regression coefficient of yields on time. This information is shown in Chart III. The design of the diagram is similar to that of the first one. The upper part deals with the years 1900 through 1915, the lower with the years since then. The irregular solid line shows the course of business activity. The dashed line is this

CHART III



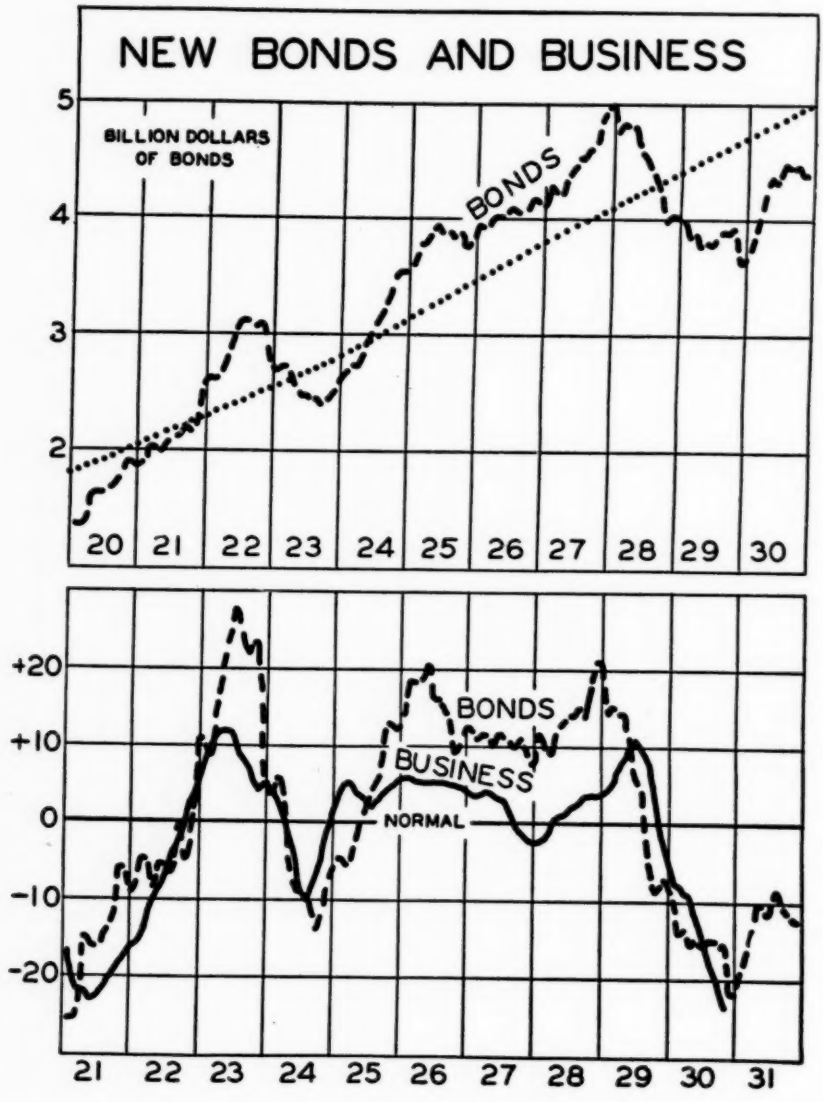
measurement of the rapidity of rise or fall in the security markets.[1] As long as this line remains above the zero level it signifies that the markets are rising, and the higher above, the faster the rate of rise. When below the zero level it signifies a downward trend in market prices. Since 1919 the measurement is based on both stock and bond markets, prior to that time on bond markets alone. There is apparent some degree of relationship which becomes better in the years since the War. Furthermore, just as in the case of the interest rate line in Chart

[1] The data represented by the dashed line are the simple (curvilinear) regression estimates of business secured from this rate of change in security yields.

I, the curve for the rate of change in security prices in this diagram has been bodily moved forward one year. Although the degree of correlation in this instance is not so great as in the case of interest rates it nevertheless seems sufficiently important to justify the inclusion of this series in a forecasting index for business.

The fourth and final series which was included in the forecasting index

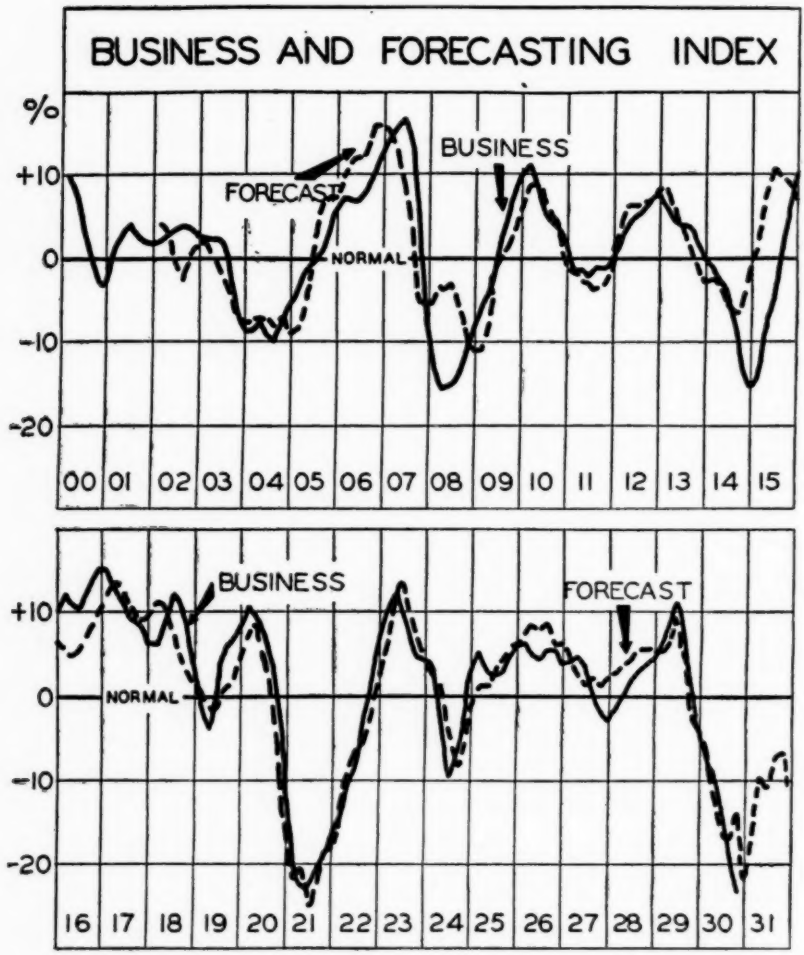
CHART IV



is based on a direct measurement of the amount of new long-term bond flotations. These data are not available for the early years, nor is any one who works with them entirely satisfied that they are as accurate as they should be for these purposes. Nevertheless, such as they are, they do show some forecasting properties, which is, of course, to be expected in the light of our hypothesis as to how these business fluctuations are produced. In Chart IV there is once more an upper and a lower part. In the upper part the irregular dashed line represents the twelve-month trailing total of new corporate and municipal long-term bond flotations in the United States, excluding those of investment trusts. The smoother, gradually rising, dotted line is the trend. In the lower part of the diagram the percentage deviations of these flotation data from their trend are represented by the irregular dashed line. The irregular solid line represents the course of business activity. A rather unusual similarity of fluctuation is apparent. As in earlier instances it is to be noted that the flotations data move one year in advance of the business curve. The dates at the bottom of the diagram apply to the business curve, while those half-way down apply to the flotations figures. Evidently this is another series which we cannot afford to omit from our forecasting index for business.

We have now discovered four sets of statistical measurements which from the statistical point of view seem to have a bearing on the condition of business one year later, and which from the economic point of view could reasonably be expected so to have. Their forecasting properties seem to be real rather than accidental and the reasons for these properties logical rather than spurious. The next step is to combine them into a single forecasting index in the hope that this will yield a more accurate prophecy than can be obtained from the individual series. The weights selected for combining the several series were naturally those which would result in the most accurate index of subsequent business conditions. The correlation technique was employed and since there was nothing unusual or unique about the computations, their description will be omitted. The final forecasting index is shown in Chart V. As in earlier instances there is an upper and a lower part. The upper part deals with the years 1900 through 1915 and the lower with the later years. The irregular solid line represents again the course of business activity. The dashed line represents the forecasting index. A rather exceptional degree of agreement is apparent. As in former cases the dashed curve has been moved forward one year so that what is shown in the diagram in the 1930 space, for example, is based on data computed in 1929. The forecasting index fluctuates one year in advance of the business curve, so that by moving

CHART V



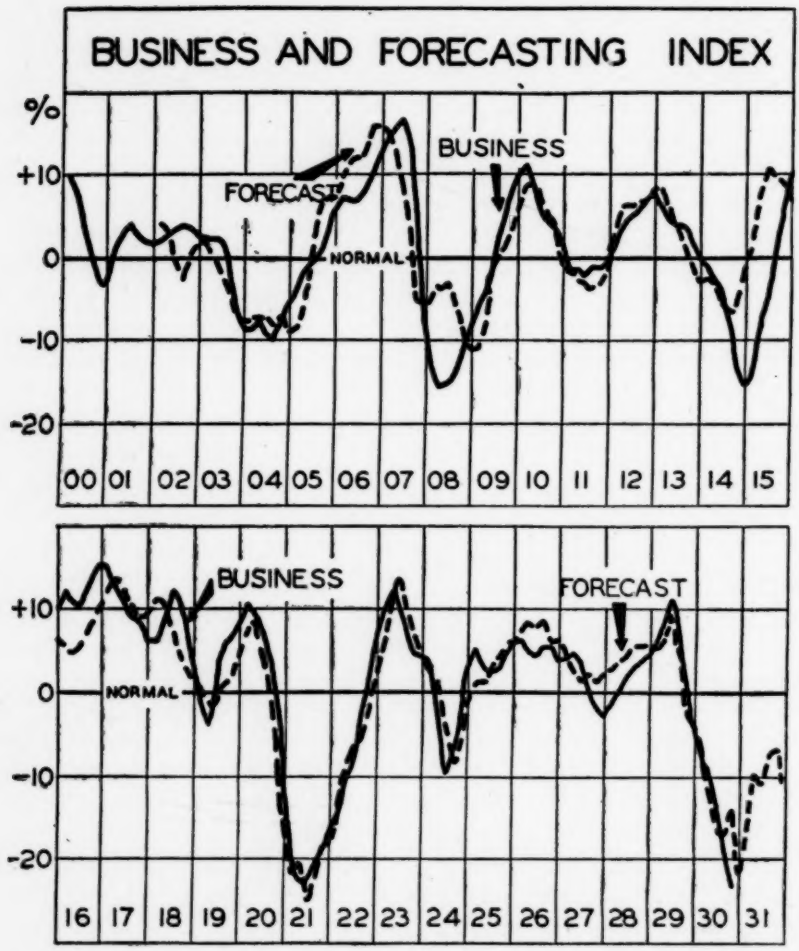
the forecasting curve forward one year the two are brought into agreement.

It would be interesting to discuss the claims that can be made for and against this index, the kinds of factors that it is necessary to keep in mind in interpreting it, the qualifications that must be made in using it, the light which it sheds on the functioning of the business cycle and the avenues of further research which the work so far has suggested as profitable. However, there is space for but a very few comments. According to the accepted statistical tests the accuracy of the index is exceptional. The correlation between the two curves as shown is .87

for the entire thirty-year span. Omitting the year following the opening of the World War, which can scarcely be considered a fair test, the correlation is .91. For the last eleven years where more accurate and desirable data have been available the correlation is .96. This agreement between the two is equal to the average agreement between the generally accepted business index numbers taken concurrently. The correlation during the last decade between the business index of the Standard Statistics Company and that of the American Telephone and Telegraph Company is, for example, approximately .94. Thus the forecasting index forecasts business a year in advance about as accurately as these two great institutions measure it currently. The economic hypothesis and the statistical results seem mutually to support each other. The hypothesis seems justified in the light of the statistical results and the statistical procedures find a reasonable and logical explanation in the hypothesis. On the other hand, there should be definitely and emphatically stated the warning which the writer has always given when discussing these kinds of forecasters.[1] This is that a constant and vigilant watch must be maintained to assure oneself that the reasons, both statistical and economic, why the forecasts have been accurate in the past are unimpaired in the present. It should be remembered that the index runs one full year ahead of business and in the space of that time economic accidents, such as war, could develop. The index does not contemplate the prophecy of economic accidents. Furthermore, the month-to-month fluctuations in the index are not to be regarded as of much significance. The index does forecast the general level of business one year in advance, but it can scarcely be expected that it will also indicate accurately the minor monthly fluctuations that far ahead. A good case could be made, for example, for a recovery of business in the months immediately ahead of us which would be less rapid than that indicated by the index. On the whole, however, we are justified in concluding that the index does carry us one step further in the fascinating search, commenced many years ago, to find something which does actually and accurately forecast the course of general business activity.

[1] See this JOURNAL, December, 1927; and PROCEEDINGS, March, 1929.

CHART V



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[1] See this JOURNAL, December, 1927; and PROCEEDINGS, March, 1929.

THE TEST OF SIGNIFICANCE FOR THE CORRELATION COEFFICIENT

By Egon S. Pearson, *University of London*

In applying the methods of statistical analysis it is generally our aim to discriminate between two or more alternative hypotheses regarding the factors which have controlled certain observed events, which form what we term a sample, or samples. If the process is examined in a little detail it will be found that the procedure may be described as follows:

(a) We define a hypothesis to be tested.
(b) We choose the criterion (or criteria) whose numerical value, derivable from the observations, is most suitable for testing the hypothesis. In doing this we recognize that the criterion is not a single valued expression even if the hypothesis be true, but will vary from one sample of observations to another.
(c) We therefore refer the observed value of the criterion to this sampling distribution—e.g. to a normal probability scale, a χ^2 probability scale, etc.—and so obtain a measure of the likelihood of the hypothesis.
(d) Finally, if judged on this probability scale the observed criterion is not exceptional, we conclude that upon the information available there are no grounds for discarding the hypothesis; or if the value prove exceptional we consider the possibility of alternative hypotheses.

The step (a) requires in general only a little careful thinking, but (b) leads us into an interesting field at present not fully explored. Here we must first decide upon the principles by which to determine what we mean by a "suitable" criterion. The importance of this problem seems first to have been realized by R. A. Fisher;[1] it has also been considered from a slightly different aspect by J. Neyman and the present writer.[2] When (b) has been settled it becomes a question of mathematics, which may or may not be within our powers of solution, to determine the sampling distribution of the criterion.

These steps in reasoning may be illustrated in the case of the test for significance of the product-moment correlation coefficient between

[1] R. A. Fisher, *Philosophical Transactions of the Royal Society*, A. Vol. 222, pp. 309–368.
[2] J. Neyman and E. S. Pearson, *Biometrika*, XX A, pp. 175–240 and pp. 264–294. *Bulletin of the Polish Academy*, Krakow, March 1930, pp. 73–96.

two variables x and y. Let it be assumed that the factors controlling the variation in x and y would lead in the aggregate to some one or other of the set of probability relations,

$$df=\frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}}e^{-\frac{1}{2(1-\rho^2)}\left\{\frac{(x-\bar{x})^2}{\sigma_x^2}-\frac{2(x-\bar{x})(y-\bar{y})}{\sigma_x\sigma_y}\rho+\frac{(y-\bar{y})^2}{\sigma_y^2}\right\}}dxdy \qquad \text{(i)}$$

where $\bar{x}$, $\bar{y}$, σ_x, σ_y and ρ are undetermined. In other words, suppose that the population distribution is *some* normal bivariate surface. Then the most suitable criterion for testing the hypothesis that $\rho=0$ from a sample of n pairs of values (x, y) is the sample product-moment coefficient of correlation r. Further, the sampling distribution of r if the hypothesis be true is known to be,

$$df=\frac{\Gamma\left(\frac{n-1}{2}\right)}{\sqrt{\pi}\,\Gamma\left(\frac{n-2}{2}\right)}(1-r^2)^{\frac{n-4}{2}}dr. \qquad \text{(ii)}$$

If, however, law (i) is not applicable, the test may be invalidated for two reasons,

(a) r may no longer be the appropriate criterion with which to test the hypothesis of no association,

(b) The sampling distribution of r may no longer follow law (ii). The problem has been discussed recently in this JOURNAL by George A. Baker [1] who has indicated clearly the danger involved in assuming without evidence that the "normal theory" law is applicable when the two variables are not normally distributed. He has pointed out that the work of Soper [2] in 1913 suggested that to the order of terms in n^{-2} the standard error of r was independent of the population form. Soper did not, however, deal with the higher moments, nor did he obtain the next term in the approximation to σ_r, so that it is not possible to judge from his work the adequacy of the first two terms when n is small.

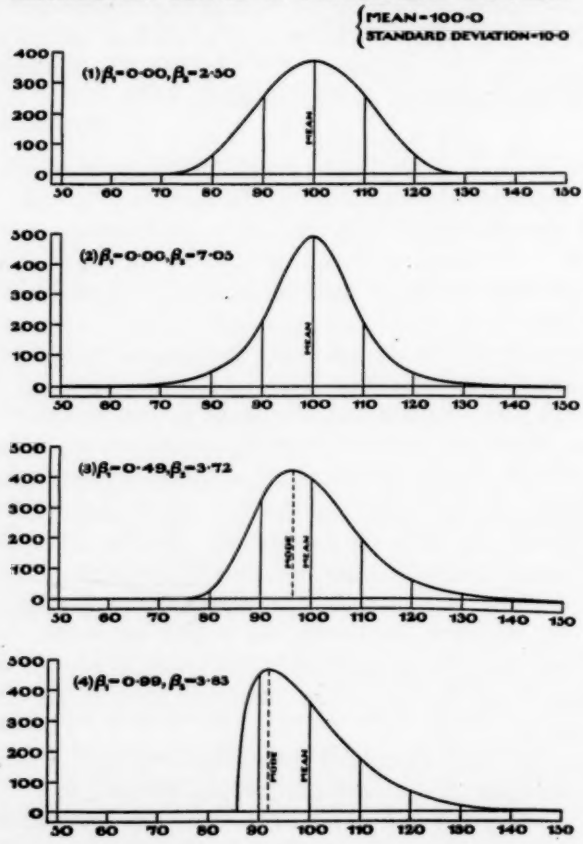
Dr. Baker has further explored the position with some experimental sampling, and finds in the case that he has chosen, that r no longer follows the distribution (ii). He has, however, considered a very extreme case of skew variation where I think one would naturally hesitate to use the coefficient of correlation as a measure of association, and it is important, therefore, to obtain a little fuller information as to the limits within which the probability scale associated with equation (ii) may still be safely used. The problem of determining satisfactory

[1] G. A. Baker, this JOURNAL, December 1930, pp. 387–396.
[2] H. E. Soper, *Biometrika*, IX, pp. 91–115.

alternative criteria when r becomes inefficient is probably not a simple one, but it is at any rate possible to obtain experimentally some appreciation of how sensitive the sampling distribution of r is to changes in population form.

For this purpose I have chosen from the population distributions used in other experimental work [1] the four for which the frequency curves reduced to a standard scale are shown in the accompanying diagram.

DISTRIBUTION CURVES OF POPULATIONS SAMPLED.



These curves are as follows:

(1) A Pearson Type II curve; $\beta_1 = 0.00, \beta_2 = 2.50$
(2) " " Type VII " $\beta_1 = 0.00, \beta_2 = 7.05$
(3) " " Type III " $\beta_1 = 0.49, \beta_2 = 3.72$
(4) " " Type I " $\beta_1 = 0.99, \beta_2 = 3.83$

[1] (a) E. S. Pearson and N. K. Adyanthāya, *Biometrika*, XX A, pp. 356–360; XXI, pp. 259–286. (b) "Sophister," *Biometrika*, XX A, pp. 389–423.

The proportionate areas beyond certain deviations from the mean are also compared in Table I with those for the normal curve. Such

TABLE I
SHOWING PROPORTIONATE TAIL AREAS BEYOND CERTAIN DEVIATIONS FROM THE MEAN

Deviations from mean in terms of standard deviation	Populations				
	Normal	(1)	(2)	(3)	(4)
	Further proportions as for + deviations				
−2.5	.006	.002	.012	.0000	.....
−2.0	.023	.018	.025	.0033	.....
−1.5	.067	.070	.056	.038	.0000
−1.0	.159	.172	.127	.153	.147
+1.0	.159	.172	.127	.162	.160
+1.5	.067	.070	.056	.080	.090
+2.0	.023	.018	.025	.038	.047
+2.5	.006	.002	.012	.017	.022
+3.0	.0013	.0000	.0059	.0075	.0095
+3.5	.0002	.....	.0032	.0030	.0035
+4.0	.0000	.....	.0018	.0013	.0011
+4.5	.....	.....	.0011	.0005	.0003
+5.0	.....	.....	.0006	.0002	.0000

a series is of course not in the least exhaustive, but may be taken as typical of various forms of non-normal variation met with in practical experience. From each of these populations I have drawn repeated samples of (a) 10, and (b) 20, independent pairs of values (x, y), and found in each case the product-moment coefficient of correlation, r. Two hundred and fifty samples were drawn in every case except that for population (3), 395 samples were available in the case of $n=20$. Ten pairs of values will be rarely used in practice for calculating a correlation coefficient, but this case and that of 20 were chosen as likely to provide very crucial tests of the adequacy of "normal theory." The results of the experiment are given below.

In Table II the experimental and "normal theory" values of the

TABLE II
FREQUENCY CONSTANTS IN EXPERIMENTAL DISTRIBUTIONS OF r

Populations sampled	Samples of 10				Samples of 20			
	σ_r	B_1	$\sqrt{B_1}$	B_2	σ_r	B_1	$\sqrt{B_1}$	B_2
(1)	.3234	.000	−.000	2.284	.2413	.032	+.178	2.710
(2)	.3487	.000	+.011	2.451	.2405	.005	+.074	2.849
(3)	.3238	.007	−.082	2.620	.2289	.038	+.196	2.726
(4)	.3181	.000	+.007	2.481	.2308	.025	+.160	2.564
Normal theory values	.3333	0	0	2.455	.2294	0	0	2.714
Standard errors *	.0127	....	.105	.147	.0095	....	.125	.204

* Standard errors are those appropriate for 250 samples.

frequency constants are compared; the latter have the following values obtained from equation (ii)

$$\left.\begin{aligned} &\text{Mean } r=0; \qquad \sigma_r=\frac{1}{\sqrt{n-1}}; \qquad B_1=\frac{\mu_3^2}{\mu_2^3}=0; \\ &B_2=\frac{\mu_4}{\mu_2^2}=3-\frac{6}{n+1}; \qquad B_4=\frac{\mu_6}{\mu_2^3}=\frac{15(n-1)^2}{(n+1)(n+3)} \end{aligned}\right\} \quad \text{(iii)}$$

It will be seen that the differences between the observed and theoretical values are never large compared with the standard errors. These latter have been calculated approximately as follows:

for a standard deviation $=\frac{1}{2}\sigma_r\left\{(B_2-1)/N\right\}^{\frac{1}{2}}$

for $\sqrt{B_1}=\left\{(B_4-6B_2+9)/N\right\}^{\frac{1}{2}}$

for B_2, taken from K. Pearson's *Tables for Statisticians*, Table XXXVIII, where N is the number of samples on which the results are based.

The most significant difference is that for $\sqrt{B_1}$ in samples of 20 from population (3). Here the difference between +.196 and 0 is 1.96 × standard error.[1] The remaining differences are all less than 1.5 × standard error. It will be seen that in the case of populations (3) and (4) the skewness ($\sqrt{B_1}$) is insignificant for $n=10$, but may be just significant for $n=20$. These are the two cases most nearly approaching the example of Dr. Baker, where 50 samples of 40 showed a definite positive skewness in the distribution of r. My values of σ_r are, however, in these cases in remarkably close agreement with the "normal theory" value of .2294. Another possible trace of divergence may be noted in the two values of σ_r in samples from the extremely leptokurtic population with $\beta_1=0.0$, $\beta_2=7.0$; these are both greater than the "normal theory" values but by amounts only just over the standard errors. Beyond this it is difficult to pick out any systematic divergence associated with either increasing skewness or kurtosis.

The sampling distribution (ii) is symmetrical, and the practical test consists in determining the chance that $|r|$ would have a value greater than the observed in random samples from uncorrelated material. I have therefore "doubled over" my experimental distributions of r (in any case the values of $\sqrt{B_1}$ in Table II show there is really no significant asymmetry) and compared them with the corresponding "normal theory" distributions of equation (ii) in Tables III and IV. The results of applying the (P, χ^2) test for goodness of fit are shown at the bottom of the tables; the agreement is very satisfactory throughout. Certain possible tendencies for divergence are of course suggested; for example

[1] This is the case where $N=395$, and standard error of $\sqrt{B_1}=.100$.

TABLE III

EXPERIMENTAL SAMPLING DISTRIBUTIONS OF r WHEN $\rho=0$ (SAMPLES OF 10)

r	"Normal theory" frequencies	Populations sampled			
		(1)	(2)	(3)	(4)
.00–.05	27.3	27	34	30	33
.05–	26.8	30	18	29	22
.10–	26.1	19	29	24	30
.15–	24.9	24	20	28	23
.20–	23.4	19	21	26	25
.25–	21.6	29	24	17	22
.30–	19.5	27	13	15	16
.35–	17.4	16	21	18	25
.40–	15.0	18	18	24	15
.45–	12.7	12	10	12	10
.50–	10.4	5	13	6	9
.55–	8.3	12	9	9	6
.60–	6.1	5	2	2	6
.65–	4.5	5	10	5	4
.70–	2.9	1	3	..	2
.75–.80	1.8	..	4	1	..
Greater than .80	1.3	1	1	4	2
Totals	250.0	250	250	250	250
Goodness of fit	n_1	13	13	13	13
	$P(\chi^2)$	0.31	0.14	0.49	0.74

when $n=10$ there appear to be too few high values of r for the platykurtic population (1) and too many for the leptokurtic population (2). But as the results are based on only 250 samples this difference may

TABLE IV

EXPERIMENTAL SAMPLING DISTRIBUTIONS OF r WHEN $\rho=0$ (SAMPLES OF 20)

r	"Normal theory" frequencies	Populations sampled				"Normal theory" frequencies
		(1)	(2)	(4)*	(3)*	
.00–.03	25.0	22	32	21	39	39.5
.03–	24.6	24	18	34	38	38.9
.06–	23.9	29	23	23	37	37.8
.09–	22.9	22	25	24	41	36.1
.12–	21.6	23	19	14	28	34.2
.15–	20.1	14	20	19	34	31.7
.18–	18.3	16	18	17	38	29.0
.21–	16.5	18	14	15	27	26.1
.24–	14.7	12	12	14	19	23.1
.27–	12.7	12	11	15	16	20.1
.30–	10.8	14	11	7	16	17.1
.33–	9.1	8	9	14	10	14.4
.36–	7.5	10	10	11	13	11.8
.39–	6.0	4	3	6	10	9.4
.42–	4.7	6	12	4	10	7.4
.45–	3.6	3	4	5	8	5.6
.48–	2.6	2	3	3	5	4.2
.51–	1.9	5	3	3	2	3.0
.54–	1.3	3	1	..	..	2.1
.57–.60	0.9	2	..	..	1	1.4
Greater than .60	1.3	1	2	1	3	2.1
Totals	250.0	250	250	250	395	395.0
Goodness of fit	n_1	15	15	15	17	.....
	$P(\chi^2)$	0.85	0.55	0.54	0.83	.....

* Note that the columns for populations (3) and (4) are interchanged in order to bring the former (which contains 395 samples) against appropriate "normal theory" values in the margin.

not be real. The experimental method is too crude to detect differences of a fine shade unless the number of samples is very greatly increased, and there is little doubt that if the exact mathematical law of distribution of r were obtained such differences would be found to exist, particularly towards the tails of the curves. But the conclusions which are drawn on practical statistics do not generally need to be based on a very fine probability scale; we find that this deviation would occur through chance in about 5 per cent of samples, a second in only 1 per cent, etc., and draw our inferences accordingly. For example for $n=10$, "normal theory" tells us that in 250 random samples from uncorrelated material we should expect to find 10.5 with $|r| \geq .65$. We find in the experiments

for Population cases:	(1)	(2)	(3)	(4)
	7	18	10	8

While for $n=20$ we should expect in 250 samples 11.6 with $|r| \geq .45$. We find

for Population cases:	(1)	(2)	(3)	(4)
	16	13	12 (reduced from 395)	12

It is only the second value in the row for samples of 10 which appears a little exceptional. Taken as a whole the extent of agreement has proved considerably greater than was anticipated before the experiment was commenced, and it seems to indicate that the coefficient of correlation is another of the criteria based on *ratios*,[1] whose distribution even in very small samples is remarkably insensitive to changes in the form of the population.

In the illustration discussed above the population distributions of both variables, x and y, have been taken the same; it is hoped shortly to compute the values of r in cases where the two distributions are different—e.g. where x is drawn, say, from population (1) and y from (4). It is possible that this change may have some effect on the resulting r-distribution.

The case of the distribution of r where ρ is not zero is more complicated. For here both n and ρ and the population forms may vary. But a recent investigation undertaken by the author suggests that even here the distribution of r is remarkably stable.[2]

[1] This point has arisen in connection with some of the forms of the test for the analysis of variance, examined experimentally in the papers referred to in footnotes [1](a) on p. 130 and [2] on p. 134.

[2] E. S. Pearson, *Biometrika*, XXI, pp. 356–360.

CAUSES OF BIRTH RATE FLUCTUATIONS[1]

By Harold Hotelling and Floy Hotelling, *Stanford University*

The causes of changes in birth rates are of fundamental importance for all problems involving the size of a human population. A long, slow decline in the birth rates of western countries has occasioned many comments, and has been ascribed by various writers to a multitude of causes.

Whatever statistical evidence has been adduced for the diverse theories amounts chiefly to showing that some condition which obtained in a by-gone era of high birth rates is no longer in existence. Logically this is weak. Too many things have changed. There are not enough independent cases.

But possibly underlying causes can be more clearly discerned if we can analyze the short-term changes, the frequently repeated increases and decreases, which provide the considerable number of cases upon which statistical inference can be based.

Certain periods of extreme variability in birth rates seeming to occur in two-year cycles were first noted by Yule in 1906, in his study of births and marriages in England and Wales.[2] Basing his study on annual rates, Yule discovered two such periods of violent oscillation in England's birth rate, namely, 1868–76, and 1889–96. He also referred to a similar period of jaggedness in the birth rate of Holland in 1857–67, and to the "curious fact that 1891 and 1893 were years of maximum, and 1892 of minimum birth rate in nearly every country of Europe."

These same periods were again noticed by Dorothy S. Thomas in 1925.[3] She also pointed out a "patch of two-year periodicity" in 1905–08.

Chart I shows similar up and down fluctuations in 1846–50, 1852–56, 1883–88, and 1911–13.

The present study offers answers to both of Yule's queries: What were the causes of the widespread variability in birth rates during these periods? and Why did they seem to result in two-year cycles?

[1] Revision of a paper delivered at the Ninety-Second Annual Meeting of the American Statistical Association at Cleveland, Ohio, December 31, 1930.

[2] G. U. Yule, "Changes in Marriage and Birth Rates in England and Wales During the Past Half-Century," *Journal of the Royal Statistical Society*, Vol. LXIX, March, 1906, pp. 100–132.

[3] D. S. Thomas, *Social Aspects of the Business Cycle*, London, 1925, p. 97.

The results of this study appear to show that the extremely great fluctuations of births at these abnormal periods were due principally to world-wide epidemics of either cholera or dysentery, small pox, or influenza.

The seeming two-year cycles to which Yule first called attention, we find are only such as might appear by chance, so far as tests for periodicity go. Upon applying three distinct tests, we find no significant cycle in numbers of births. None of these diseases occurs in anything like a two-year cycle. However, because these epidemics often happened at irregular intervals for several years in succession, they simulated a wave-like motion on account of the troughs of births which they always caused, followed by corresponding peaks.

Actual numbers of births, deaths, and marriages in England and Wales were chosen for study, rather than rates, which, except in census years, are only approximations. Furthermore, in order to study the fluctuations more closely, it was decided to use quarterly instead of annual figures. The data were taken from photostats of original records dating from 1838, the first year of compulsory civil registration, until the present time; these were kindly furnished by the office of the Registrar-General.

The last of the eight periods under discussion having occurred in 1911–13, the plotted curves of the data extend only until 1914, thus eliminating the war and post-war years with all their abnormal complications.

Seasonal variation was eliminated from the death curve by the well-known method of dividing each quarterly figure by an index number calculated by means of median link relatives. The fitted trend line for deaths was taken to be of the third degree, since coefficients of a higher degree were not significant. The criterion of significance we applied was that the probability of a greater value by pure chance should be less than a small quantity, which we took conventionally to be .01.

A new method for taking account of seasonal variation and of secular trend at the same time was employed in plotting the curves for marriages and births. As the data were given by quarter-years, it was only necessary to calculate the trend line for each quarter using the actual numbers of marriages or of births, and then to plot the deviations from trend in a composite graph (Chart I). Not only does this method take less time for calculation than the ordinary link-relative method, but it makes definite estimates of variance possible.

Fourth degree trend lines were fitted to the quarterly marriage data. Straight trend lines were fitted to the quarterly births from 1838–75, curves of the third degree being necessary to fit the data from 1876–

1914. That the results are good is shown by the fact that the serial correlation of deviations in one quarter with those in the next is practically zero. For instance, between the first and second quarters in the second half of this period, it was only .03278.

It is apparent from the graph that the variance (i.e. the squared standard error, or the mean value of the square of the deviations from trend) of births was greater in the period 1876–1914 than in the period 1838–75. The total variance in the first period, after elimination of trend and seasonal influence, amounted to 33,182,500 "squared births"; that in the second period, to 51,615,000 "squared births," or 56 per cent more than that in the first 38 years. The standard deviation in the first period was 5,760 births each quarter-year; in the second period, it was 7,184 births each quarter-year.

Some of the difference in variance between the two periods may have been due to a greater population in the second period. But the increase of variance when the data for each quarter are considered separately is very much greater in the second and fourth quarters than in the first and third. Table I contains these comparisons.

TABLE I

VARIANCES IN BIRTHS OF ENGLAND AND WALES, 1838–1914

Quarter	Total variance in first half of period	Total variance in second half of period	Increase of variance second over first half
			Per cent
I	80,310,000	93,540,000	16
II	16,140,000	53,660,000	232
III	14,920,000	17,090,000	15
IV	21,360,000	42,170,000	97
Average	33,182,500	51,615,000	56
Standard deviation	5,760	7,184	25

The number of births in the second quarter often seemed diminished by an epidemic of cholera or similar diseases in the third quarter preceding; the number of births in the fourth quarter was similarly often diminished by an epidemic of influenza in the first quarter of the same year. The number of both kinds of epidemics was much greater in the second half of this period, and the number of cholera epidemics was greater than the number of influenza epidemics. Of cholera or dysentery epidemics, there were 12 in the first period, and 17 in the second; of influenza epidemics, there were 7 in the first period, and 11 in the second.

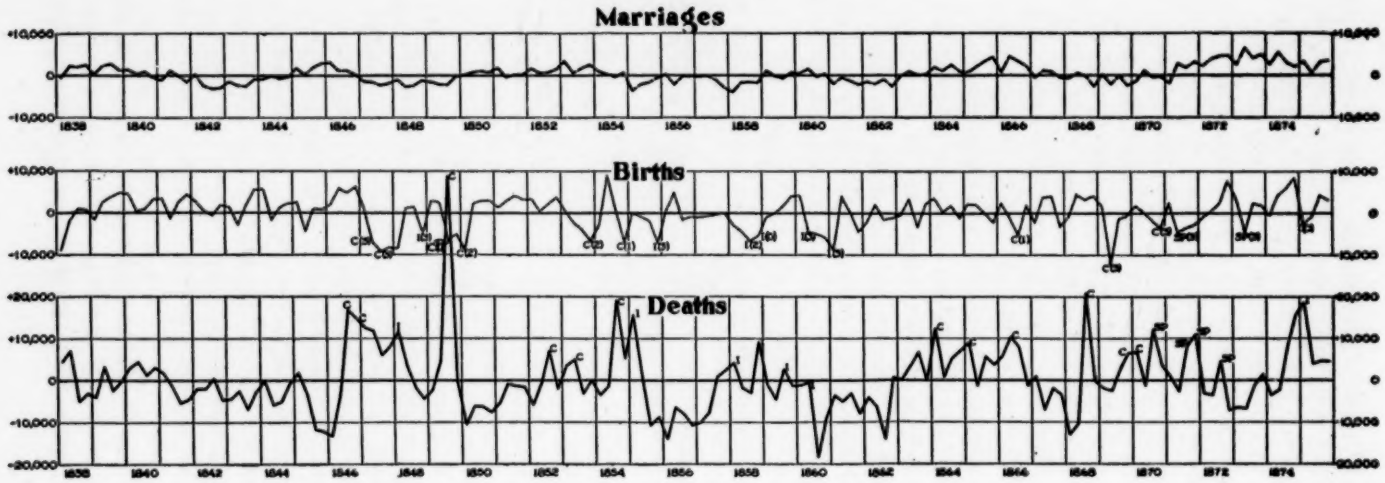
How much of the variance in births was due to possible random sampling error? A little computation demonstrates that the deviations of births from trend are far greater than are to be expected on the

CHART I

BIRTHS, DEATHS AND MARRIAGES IN ENGLAND AND WALES

Secular trend and seasonal variation eliminated.

1838–1875



1876–1914

Marriages
Births
Deaths
I - Influenza. C - Cholera, Dysentery, etc. S.P. - Smallpox. ~ Numbers indicate quarters since last epidemic.

basis of chance. In the fourth quarter of 1890, for instance, the number of births reached the low point 203,500, or 17,000 below the trend. The well-known formula npq for the variance to be expected by chance can never give a greater result than np, the mean number of occurrences. This mean number of occurrences in the fourth quarter of 1890 would be the ordinate of the trend line, 220,500. The standard error must be less than the square root of this number, approximately 470. The deviation 17,000 must therefore be more than 36.2 times its standard error. The probability of so great a deviation by chance is incredibly small.

In like manner, it can be shown that the deviations of births from trend in the other seven abnormal periods set forth on the graph are extremely significant in size. Evidently, therefore, only an infinitesimal part of the variance in births may be ascribed to errors of random sampling.

Then, the question naturally arises, could not these abnormalities in the numbers of births be explained by deficiencies or increases in the numbers of marriages three or four quarters before?

In 1914, one-third of the marriages unprejudiced by prenuptial conception in Australia, and one-fourth of those in New Zealand, resulted in births 9 to 12 months later, while the proportion was probably larger in earlier years.[1] In spite of the lack of data as to the relationship in England and Wales between variations in births and in marriages 9 to 12 months before, it is reasonable to suppose that a deficiency in the number of marriages in any quarter might explain an abnormal downward departure from trend in the numbers of births 3 or 4 quarters later. Conversely, a great surplus of births above the trend might be due to a corresponding increase in marriages 3 or 4 quarters before.

Whatever deviations from trend in births there were, however, due to a deviation from trend in marriages 9 to 12 months previous, must necessarily be smaller in variance than the number of marriages, since about one-sixth of all marriages are sterile,[2] and since another large portion of them result in births either earlier or later than 9 to 12 months after consummation.

But a glance at Chart I shows that the amplitude of the birth deviations is very much larger than that of the marriage deviations. How much, then, of the birth variance may be attributed to variance in marriages?

[1] C. Gini, S. Nasu, R. R. Kuczynski, O. E. Baker, *Population*, Chicago, 1930, p. 70.

[2] C. J. Lewis and J. N. Lewis, *Natality and Fecundity*, Edinburgh, 1906, p. 152; A. J. Lotka, "Sterility in American Marriages," *Proceedings of the National Academy of Sciences*, Vol. 14, 1928, p. 109; A. Newsholme and T. H. C. Stevenson, "Decline of Human Fertility in the United Kingdom and Other Countries as Shown by Corrected Birth Rates," *Journal of the Royal Statistical Society*, Vol. LXIX, 1906, p. 62.

In order to measure this quantity, we first calculated the coefficient of correlation of deviations from trend of births with those of marriages in the third preceding quarter, and then with those of marriages in the fourth preceding quarter. The coefficient of correlation in the first case, using our data for 77 years, we found to be .1654, and in the second case, .1504. The multiple correlation[1] was .2617. It is thus apparent that although a positive correlation exists between the variance in births and that in marriages 6 to 15 months before, it is not large.

The proportion of the variance in births due to the variance in marriages in the third and fourth preceding quarters is approximately equal to the square of the coefficient of correlation between the two, namely .0685. In other words, only 7 per cent or about one-fourteenth of the variance in births can be ascribed to a variance in marriages 6 to 15 months before.

This leaves 93 per cent of the variance in births to be accounted for in some other way.

Yule and Thomas have already concluded that economic conditions had nothing to do with the apparent two-year cyclical fluctuations in the birth rates of England and Wales. Both of these investigators found that the birth rate in general responds much more slowly to economic conditions than does the marriage rate. Hexter found, using Boston data,[2] that the relationship between marriages and births is far stronger than that between employment, his index of economic conditions, and births. Economic conditions appear to have their chief effect through postponement of marriages, and we have just seen that variations in marriages in the third and fourth preceding quarters account for only 7 per cent of the variance in births. Even if the total effect of economic conditions on births were three times the effect of postponement of marriages for all causes—a very liberal estimate—we should still have about 80 per cent of the variance which must be due to some other cause.

Perhaps some of the variance in births may have been due to the absence from the country of great numbers of men at time of war or threat of war who might otherwise have become fathers. But, in this period of English history, aside from intermittent wars of conquest in Asia and Africa which usually involved only the regular standing army and navy of England, the Crimean War, 1853–56, and the two Boer Wars, 1880–81, and 1899–1902, were the only wars which drew away

[1] This value would be exceeded by chance in very slightly more than 5 per cent of cases if there was no real correlation.

[2] M. B. Hexter, *Social Consequences of Business Cycles*, New York, 1925, p. 74.

from England large numbers of volunteer fighters. These, of course, were usually young, unmarried men; consequently, the chief effect of such wars upon the birth rate would come about by a diminution of marriages during the war, compensated for by an increase of marriages both before and after the war. As we have seen, even such a variance in marriages could cause at most only 7 per cent of the variance in births. Therefore, we must conclude that this part of the variance is very small.

The graph (Chart I) indicates, moreover, that the oscillations of the birth curve at all of the war and threat-of-war periods were not nearly so great as in the period 1886–96 when England was engaged in no war at all. Similarly, the variance in births in 1847, 1849, 1858, 1860–61, and 1869 occurred at times when absence of troops from home could not possibly have explained it. We are forced to believe, therefore, that although a small part of the variance in births during this long period may have been due to family separations on account of war-duty, especially at certain times, by far the largest part of the variance must have been caused by some other factor.

A further small part of the variance may be due to previous disturbances from trend. For instance, a diminished number of births in any year, due to a war, economic conditions, or what not, would mean that a generation later there would be fewer people than otherwise who might become parents. However, upon consideration, it must appear that the second variance would be very much smaller in amplitude than the first, and spread over a much longer period. For, in the first place, the overlapping effect of the increase of births following a depression must tend to flatten out the second variance to a large extent. In the next place, the reproductive period being very long, perhaps 30 years in all, when compared to the length of a war, the longest of which in the epoch we are studying was 3 years, or when compared to the length of an economic depression, a still greater flattening of the second variance must be expected. In fact, it is questionable whether the effect of a previous oscillation in the birth curve could be detected in a graph such as ours at all.

After making these liberal allowances, there remains the greater part of the variance in births to be accounted for by causes independent of economic fluctuations and military events. What about this major part of the variance?

The answer to this question, we believe, lies in the fluctuations in the numbers of deaths which the graph (Chart I) indicates are far greater than those of either births or marriages. The total variance in deaths after elimination of trend and seasonal variation, is 113,812,500, more

than double the variance in births. The standard deviation for deaths in the whole period is 10,668, compared to the standard deviation for births of 5,760 in the first half of the period, and 7,185 in the second half.

We found a negative correlation of −.3796 between deviations from trend of deaths and those of births three quarters later. This is distinctly greater than even the *multiple* correlation between deviations from trend of births and those of marriages in the third and fourth quarters preceding, which was .2617.

At every one of the eight periods of violent oscillation in the numbers of births, great, jagged peaks in the death curve may be observed. These death peaks are clearly described and explained in epidemiological literature as well as in the annual reports of the Registrar-General. Each of these abnormal death peaks represents a world-wide epidemic of either cholera or dysentery,[1] smallpox, or influenza.

Extreme variations in the number of births may be found in each of these epidemic periods. How may these deviations in births both above and below the trend be accounted for? A number of explanations may be advanced.

One reason may be the large number of deaths of prospective mothers at the time of an epidemic. It is generally recognized that pregnancy, especially in its later stages, is a grave risk in an attack of any of these three diseases, and that it greatly increases the likelihood of fatality. On the other hand, such an attack usually causes abortion or premature birth,[2] so that in a country like England where a birth is counted if the child has ever breathed, the number of recorded births might not be greatly affected by such deaths. As a matter of fact, stillbirths, which until recently were not recorded in England, are always in other countries greatly augmented during such epidemics.[3]

Another possible cause may be the deaths of large numbers of women in the reproductive ages 15–45, whether pregnant or not, who might have borne children if they had lived, or of men who might have become fathers. Smallpox and cholera have no predilection for any special age, but epidemic influenza is known to attack both men and women in the healthy ages of middle life proportionately oftener than persons in

[1] Since 1901, cholera and diarrhoea, which are often confused with each other, have been classified together in zymotic mortality tables prepared by the Registrar-General. But for the sake of brevity, the epidemics of these diseases we have labelled simply "cholera."

[2] *The Oxford Medicine*, Oxford University Press, 1927, Vol. IV, p. 794; Vol. V, p. 570; W. E. Wels, "Influenza Complicating Late Pregnancy," *American Journal of Obstetrics and Gynecology*, New York, 1919, Vol. 79, p. 247.

[3] *Studies on the Epidemic of Influenza*, Publications from University of Pittsburgh School of Medicine, 1919, pp. 57–8.

childhood or old age.[1] But in spite of the greater number of cases of influenza among men and women of middle life, death from any one of these diseases is more likely to occur if the patient is very young or very old.[2] Indeed deaths from influenza probably include no excessive proportion of women of reproductive age, although in the 1918 influenza epidemic, there was a great excess of deaths among men of these ages.[3]

But even if deaths among persons in the reproductive ages were the principal reason for a fall in the number of births, why should such a fall be concentrated usually into one quarter, to be followed immediately afterward by a phenomenal rise? If deaths of possible mothers fully accounted for the decrease in births, the depression should take the shape of a saucer. But the graph shows no such gradual falling off and recovery in the number of births. In almost every case the depression is shaped like an arrow-head—a sudden fall with an opposite rebound immediately afterward.

The explanation, then, for the curious effect of the epidemic upon the number of births must be, not so much in the large numbers of deaths during an epidemic, as in the widespread illness at that time.

Because the vital statistics of all England are given only by quarters, births and deaths in France given by months in *Statistique Annuelle de la France* for the years 1878–97 were chosen for detailed comparison with quarterly data from England during the same period.

Secular trend and seasonal variation were eliminated in the same manner as in the data for deaths of England and Wales, the trend line for births being of the first degree, that for deaths of the second degree. Chart II presents these data in graphic form. In this graph, as in England's, epidemic peaks of deaths and depressions in births a few months later are readily discernible.

During these twenty years there were only four instances in which an epidemic in France did not occur simultaneously in England. Besides, the four most severe epidemics in France were also the most severe in England. Apparently, then, it may be assumed that in general what was true of an epidemic in one country was also true in the other.

Let us examine first the cholera epidemics which were common to both countries during these twenty years, and then the influenza

[1] E. S. Thompson, *Influenza, an Historical Survey*, London, 1890, p. 411; O. Leichtenstern, Article on Influenza in *Nothnagel's Encyclopedia of Practical Medicine*, Philadelphia, 1905, p. 570; W. T. Vaughan, *Influenza, an Epidemic Study*, Baltimore, 1921, p. 145.

[2] A. L. Bowley, *London Life and Labour*, London, 1930, p. 216; W. H. Frost, "The Epidemic of Influenza," *Public Health Reports*, Washington, 1920, Vol. 35, p. 593; Registrar General's *Statistical Review of England and Wales*, 1927, pt. I, p. 35.

[3] L. K. Frankel and L. I. Dublin, "Influenza Mortality among Wage-earners and Their Families," *American Journal of Public Health*, 1919, Vol. 9, p. 731.

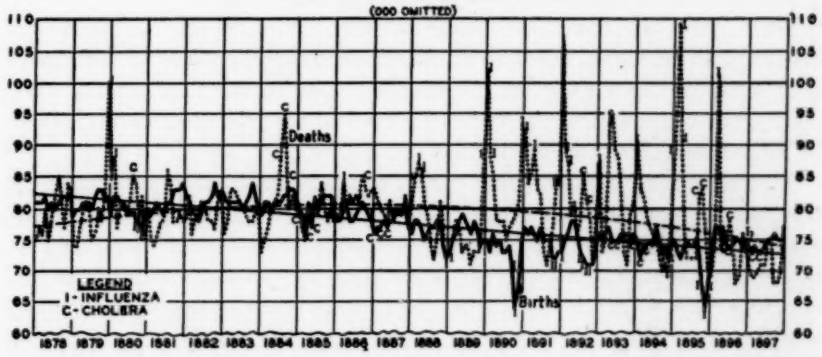
epidemics. The cholera epidemics in common occurred in 1884, 1886, 1893, 1895, and 1896. In three of these cases the epidemic in England was followed by a scarcity of births three quarters later, in two of the cases the scarcity began to show itself after two quarters.

Summarizing the five cases in France: the epidemic was followed from 6 to 8 months after it began by a defection in births which lasted until 7 to 10 months after the epidemic ceased. In all five cases, the scarcity of births was immediately followed, as in England, by a more or less marked increase in the number of births.

CHART II

BIRTHS AND DEATHS OF FRANCE, 1878–1897

Adjusted for seasonal variation



Considering now the influenza epidemics common to both countries, the depression of births in England is observed to have occurred three quarters after the epidemic peak in deaths, except in 1888, when it occurred two quarters later, and in 1890, when it began to be felt one quarter later, although the main effect in that year occurred three quarters later. The graph of births and deaths in France shows the birth defection beginning 7 to 9 months after the epidemic began, and except in 1886, when cholera interfered to obscure the effect of the influenza epidemic, the defection in births due to influenza was followed by an immediate increase in births similar to that following the cholera depression.

An analysis of the case incidence and fatality in the 1918 influenza epidemic in the United States—an epidemic which was surprisingly like all previously recorded epidemics of this disease—shows that the peak week of incidence occurs about two weeks *after* the peak week of death.[1] Consequently, the birth defection may be inferred to have taken place even less than 7 to 9 months after the peak in illnesses.

[1] E. Sydenstricker, "Variations in Case Fatality During the Influenza Epidemic of 1918," *Public Health Reports*, Vol. 36, p. 2201.

How explain this curious phenomenon in both England and France? There seems to be but one explanation. Involuntary abortions in the early months of pregnancy as a complication of the epidemic disease must have occurred in great numbers. These abortions occurring almost simultaneously were numerous enough to cause not only an appreciable decrease in the number of births a few months later, but by leaving so many women exposed to the risk of pregnancy at the same time, to cause a considerable increase in births the following year.

Medical literature confirms this supposition. Cholera, it is stated, as well as small-pox, almost invariably causes abortion. Influenza has often been observed to do so,[1] but physicians are not so positive in their statements as to this characteristic in influenza as in cholera, probably because, as this statistical study shows, abortion due to influenza occurs most often in the first month of pregnancy, while in cholera it may occur in later months. A *couche fausse* in the first month very often goes unnoticed even by the patient and would seldom, therefore, be mentioned to the physician.

This effect of influenza upon births was noticed in 1890 in France by the statistician, Turquan, and the physician, Bloch. Neither, however, mentioned abortion as a possible explanation for the birth depression. It was also noticed in the same year by two German physicians, Sperling and Stumpf. Both of them came to the same conclusion, that the deficiency was due principally to abortions during the epidemic of influenza at the first of the year. Schmid pointed out the birth depressions in Switzerland in the years 1889–95, and Jordan noticed a similar effect in the registration area of the United States in 1919.[2]

The epidemics of influenza since the War, although of much less magnitude than that of 1918, may be clearly discerned in the deviations of England's births from trend. (See Chart III.) Epidemics occurred in the first quarter of each of the years 1920, 1922, 1924, 1927, and 1929. True to form, in every case, they caused pronounced downward deviations from trend in births three quarters later.

Although Yule does not speak of it, his graphs of the birth rates of various countries show by their parallelism that the "very widespread

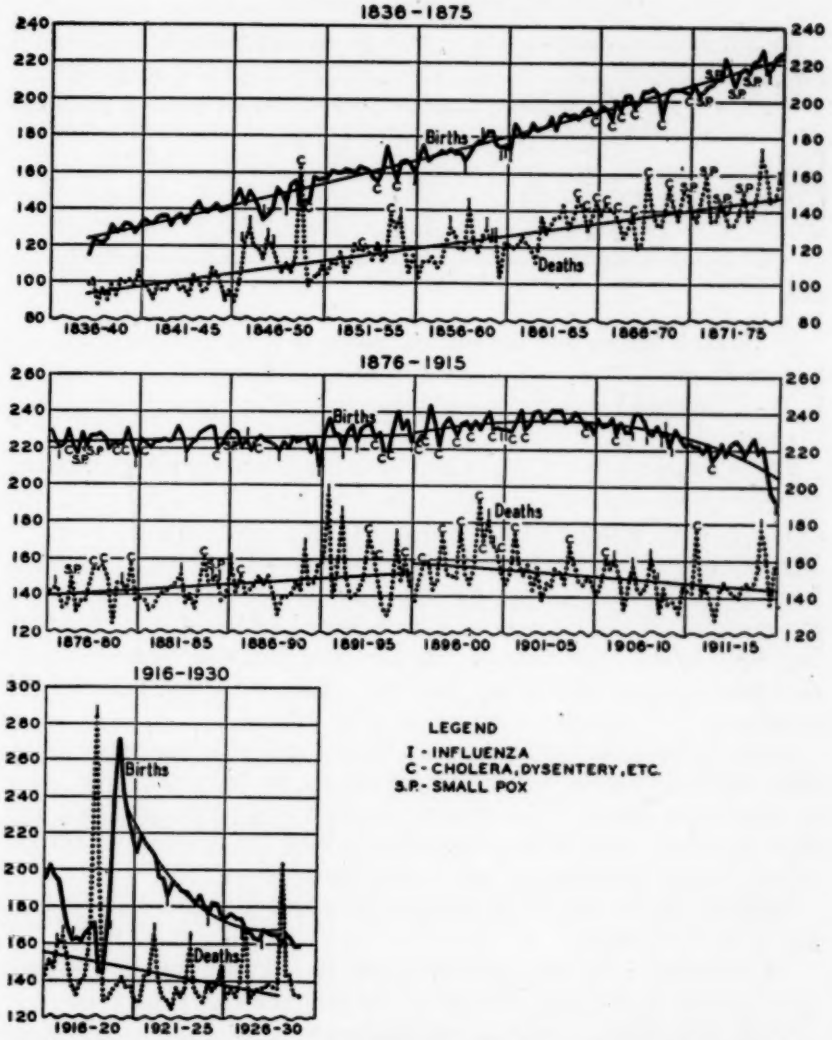
[1] *Influenza Essays*, F. G. Crookshank, ed. London, 1922, p. 437. F. Calderon, "The Influence of Influenza on Menstruation, Pregnancy, and Puerperium," *Journal of American Medical Association*, Chicago, 1919, Vol. 73, p. 982.

[2] V. Turquan, "Statistique des epidemies de Grippe de 1890 et 1892," *Journal de la Société de Statistique de Paris*, Tome 34–35, 1893–94, p. 60; A. Bloch, "Cause principale de la diminution des naissances en France en 1890," *La Semaine Medicale*, 1891, p. CCXX; P. Sperling, "Über den durch die Influenzaepidemie von 1889–90 in Deutschland verursachten Lebensfall," *Deutsche Medicinische Wochenschrift*, XVIII, 1892, p. 340; M. Stumpf, "Die Influenza-epidemie des Jahres 1889–90 und ihre Rückwirkung auf die Geburtenziffer," *Münchener Medicinische Wochenschrift*, 40, 1893; F. Schmid, *Die Influenza in der Schweiz in den Jahren 1889–94*, Bern, 1895, p. 120; E. O. Jordan, *Epidemic Influenza*, Chicago, 1927, pp. 15–16.

CHART III

BIRTHS AND DEATHS OF ENGLAND AND WALES, 1838–1929

Adjusted for seasonal variation



causes" of the 1891–93 periodicity must also have been at work in the other periods of birth fluctuations which he observed.

Coefficients of correlation between the annual birth rates of six important northern European countries 1876–1914, after eliminating a third degree trend in each case, show how much more highly correlated are countries which have much travel and trade between each other,

than are those which though adjacent to each other have little such intercourse. Table II presents these correlations:

TABLE II

CORRELATIONS BETWEEN BIRTH RATES* OF SIX NEIGHBORING EUROPEAN COUNTRIES, 1876–1914

	England	Scotland	Holland	Belgium	France	Germany
England	...	.78	.59	.28	.50	.48
Scotland	.78	...	.83	.25	.49	.42
Holland	.59	.83	...	.44	.38	.57
Belgium	.28	.25	.44	...	.57	.32
France	.50	.49	.38	.57	...	.27
Germany	.48	.42	.57	.32	.27	...

* Birth rates taken from R. R. Kuczynski, *The Balance of Births and Deaths*, New York, 1928, pp. 96–97.

In general, the highest correlations seem to exist between countries having the closest trade relations: England and Scotland; Scotland and Holland; England and Holland; Holland and Germany; Belgium and France.

The concurrent irregularities in Yule's birth rate curves of different countries were evidently consequences of world-wide epidemics moving along the usual travel routes of human beings from country to country. The velocity of propagation of the disturbance, like those of waves in water or in an elastic solid, is finite. That is why in the earlier periods the oscillations of the various curves were distinctly out of phase. But even in the early periods, Scotland and England were so closely connected by communication lines that their birth rate curves were usually, as Yule observed, in step with each other.

As to cycles in birth rates, it may be observed that the irregular recurrence of epidemics will on the hypothesis we have set forth cause alternate diminutions and increases of numbers of births in such a way that by picking out occasional intervals of four, six, or eight years from a long period, an apparent periodicity can be found.

Epidemiological literature discloses no claim to a two-year cycle in any of these diseases.[1]

The absence of a persisting tendency to anything like a two-year cycle seems to us clearly established by the following three tests:

First, periodogram analysis was applied to each of the periods mentioned by Yule in the quarterly births of England and Wales, namely 1867–78 and 1889–97, as well as to the whole period 1838–1914. Trial periods of 5, 6, 7, 8, and 9 quarters were applied to the shorter periods,

[1] A. Ransome, "On the Form of the Epidemic Wave and Some of Its Probable Causes," *Transactions of the Epidemiological Society of London*, Vol. I, 1881–82, p. 96; Sir L. Rogers, "Incidence and Spread of Cholera in India," *Proceedings of the Royal Society of Medicine*, Vol. 19, 1925–26, p. 65; M. Greenwood, "The Epidemiology of Influenza," *British Medical Journal*, 1918, p. 563.

and one of 8 quarters was applied to the whole period. The intensities found were in no case significant.

In the second place, the serial correlations of deviations of births from trend were very small.

Thirdly, a tendency to periodicity might be detected by a high negative correlation of a series with its second differences. In this case, we calculated the correlation of the series of deviations from trend of the quarterly births with their own second differences, centered a quarter later. The correlation coefficient found in this way, instead of being negative as would be expected under a hypothesis of periodicity, was +.201. Thus, there appears to be no evidence for a genuinely cyclical situation.

We conclude that since the variance of births is enormously greater than any variance that could be expected to arise from random sampling, from the effects of economic conditions as manifested through marriages, and from wars and mobilizations, some considerable disturbing element of a different nature must affect birth rates. Such an influence is found in the recurrence of epidemics which lead to profound depressions in births seven to nine months later. The importance of this factor is verified by a very detailed study of British data which has involved incidentally the introduction of a new method of eliminating seasonal and secular effects simultaneously by fitting a series of orthogonal polynomials. It is also verified by a consideration of French data.

Furthermore, high correlations observed between fluctuations of birth rates in adjoining countries can be explained on the ground of an epidemiological influence on births. The occasional periods of apparent two-year cycles can also be explained as an effect of the intermittent recurrence of epidemics.

COMMENTS ON APPLICATIONS OF RECENTLY DEVELOPED THEORY OF SMALL SAMPLES [1]

By H. L. Rietz

Both for practical and for interesting theoretical purposes, it has been recognized in recent years as important to develop special methods for the analysis of data consisting of small samples, and to learn, if possible, to what extent small samples will serve the particular purpose of establishing a certain degree of rational belief in favor of or against a certain proposition.

Previous to the present or recent period of progress with the analysis of small samples, it had been the customary procedure in dealing with sampling fluctuations of any statistic, such as an arithmetic mean, a standard deviation, a moment coefficient, a regression coefficient, or a correlation coefficient, to assume that each sample includes a fairly large number. Thus it had been customary to use probable error formulas derived on the assumption that we have a large sample. But the lower bound of large numbers remained poorly defined in this connection. For example, the usual probable error formulas have been applied to an average of as few as ten and probably to even fewer items. The chief source of error in the invalid use of probable error formulas probably lies in the fact that the statistic may not be normally distributed.

In *Biometrika* for 1908, "Student" departed from the classical procedure by investigating the distribution of certain important statistics. He dealt with a sample of n items $x_1, x_2, \ldots, x_n$ with a mean deviating $\bar{x}$ from the population mean and a standard deviation, s, calculated from the sample. His investigations are much limited, to be sure, by the assumption that the sample is drawn from a normal universe. Consider now the problem of passing judgment on the question as to whether the sample may reasonably be regarded as drawn from a normal universe with a prescribed mean, but with a standard deviation, s, known only by estimates from the sample.

"Student" found, more or less empirically, from the values of the moments that the distribution of s^2 from samples is given by a Pearson Type III curve. Although this result for the distribution of s^2 could have been derived exactly from the results of Helmert [2] for the distribu-

[1] Revision of a paper delivered at the Ninety-Second Annual Meeting of the American Statistical Association at Cleveland, Ohio, December 30, 1930.

[2] C. F. Helmert, "Ueber die Wahrscheinlichkeit der Potenzsummen der Beobachtungsfehler und über einige damit in Zusammenhang stehende Fragen," *Zeitschrift für Mathematik und Physik*, Bd. 21, 1876; *cf.* Czuber, *Beobachtungsfehler*, 1891, p. 162.

tion of the sum of squares of n errors, "Student" and others seemed to have overlooked this fact. However, "Student" showed remarkable insight and made a very significant contribution in proceeding from the distribution of s and $\bar{x}$ to the distribution of the ratio $z=\bar{x}/s$, giving us thereby an important new probability curve.

The ratio $z=\bar{x}/s$ is very important because it has direct use in passing judgment on the significance of the deviation of the mean of a sample from the corresponding theoretical or hypothetical mean. Thus, if you will think of the usual procedure after we calculate an arithmetic mean $\bar{x}$, you will recall that we ordinarily need the ratio of $\bar{x}$ to s, or to some multiple of s, before we can enter a normal probability table, to get the odds in favor of or against a discrepancy as large or larger than an assigned value. The use of $z=\bar{x}/s$ is essentially equivalent to the use of $t=\bar{x}/s_{\bar{x}}$, where $s_{\bar{x}}=s/\sqrt{n-1}$ is the standard deviation of $\bar{x}$, although the use of t is probably becoming more general than that of z because the larger "Student" tables involve t rather than z.

A still more common and perhaps more important problem in statistical practice than that of testing the significance of the deviation of the mean of a sample from a hypothetical mean is that of testing whether two samples differ significantly with respect to their mean values, or other statistics, or whether they should be regarded as belonging to the same universe.

The classical procedure in dealing with this problem was to find the probable error of each statistic, and then to accept the square root of the sum of the squares of these probable errors as the probable error of the difference of the two statistics. Then the usual interpretation was made on the basis of the implicit assumption that the ratio of the difference to its standard error taken from the samples, is normally distributed. Thus, let $x_1, x_2, \ldots, x_m$ and $x'_1, x'_2, \ldots, x'_n$ be the variates of two samples. The significance of the difference between their arithmetic means would be tested, under the classical theory, by simply calculating the arithmetic means $\bar{x}$ and $\bar{x}'$, with the probable error in the difference $\bar{x}-\bar{x}'$ and then offering an interpretation on the basis of the odds given by the normal probability integral. If we look into this procedure with some care we find that it involves the assumption that the ratio of the difference of means $\bar{x}-\bar{x}'$, to its calculated standard deviation is normally distributed. This assumption would be acceptable when the parent distribution is normal if we knew and could use the constant standard deviation of the population instead of its estimated value from the sample. The assumption is invalid for small samples when we are obliged to use the standard deviations from the sample to estimate the standard deviation of the population.

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A new procedure has been available since 1925 through the work of R. A. Fisher for small samples drawn from normal universes subject to the restriction of equal standard deviations. This new procedure is based on the fact that the ratios in question are distributed in accord with the "Student" probability curve if the samples are drawn from populations of the same standard deviation.

For the standard deviation, s_d, of the difference $d=\bar{x}-\bar{x}'$, we may write $s_d=s\sqrt{1/m+1/n}$, where

$$s^2=\frac{1}{m+n-2}[\Sigma(x_i-\bar{x})^2+\Sigma(x'_i-\bar{x}')^2].$$

The ratio $t=d/s_d$ is distributed in accord with the "Student" probability curve, provided the parent distributions are normal distributions with their standard deviations equal. These are, of course, important limitations that should not be overlooked.

"Student" published in *Biometrika*, in 1908, a brief table of his probability integral with the ratio $\bar{x}/s$ and the number of items as the arguments. In 1925, he published a much more extensive table in *Metron*,[1] with $t=\bar{x}/s_{\bar{x}}$ and the number of items as the arguments where $s_{\bar{x}}$ is the standard deviation of $\bar{x}$.

Suitable tables are thus available for testing, by means of the t-distribution, the significance of the deviation of the mean of a small sample from a hypothetical mean, and for testing the significance of the differences of two means computed from small samples, when the parent distributions are normal.

While we could easily present figures directly from the probability tables to compare the estimates of probabilities that t will exceed an assigned value by the normal table and by the "Student" table, it seems that it may mean more to many of us if we consider first a few concrete examples.

On this account, we shall present a few simple numerical applications in which comparisons are made of results from the newer theory with the corresponding results obtained in applying the classical theory developed for large samples under the assumptions that all distributions are normal.

Take first a random sample of five items. Suppose the calculated mean $\bar{x}=1$, measured from the theoretical expectation, and that the standard deviation [2] of the population estimated from the sample is $s=1$. Then $t=\bar{x}/s_{\bar{x}}=\sqrt{5}=2.236$. Turning to the "Student" table in *Metron*, we find the probability is $p=.9554$ that an $\bar{x}$ found from such a

[1] Vol. 5, No. 3, pp. 114–120.

[2] "Student's" s^2 used on page 150 equals this s^2 multiplied by $(n-1)/n$.

random sample will fall between $-\infty$ and $+1$. Then $P=2\ (1-p)$ $=.0892$ gives the probability of falling outside the range $t=\mp 2.236$.

By referring to a table of the normal probability integral, we find that the older theory would give $p=.9873$ and $P=2(1-p)=.0254$ to correspond to $t=2.236$. In terms of odds that a mean, $\bar{x}$, will deviate numerically more than 1 from theory, the situation is more striking. Thus, we should find by the older theory that the odds are 10,000 to 254, or about 40 to 1 against a deviation as large as or larger than the one observed. Under the "Student" theory, we should say the odds are 10,000 to 892 or roughly only 11 to 1 against such a deviation. We would, therefore, in this case, almost surely ascribe no significance to the "difference" by following the "Student" procedure whereas under the older practice, some statisticians might interpret the difference as significant. We should not, of course, overlook the assumption that our five items are to be drawn at random from a normally distributed population. We are likely to have very little evidence, in many practical cases, as to the validity of this assumption.

Consider next an illustration in which we are to pass judgment on the significance of the difference between the means found from two random samples. Suppose a random sample of $m=8$ items with mean value $\bar{x}_1=+1$, and standard deviation[1] $s_1=\sqrt{2}$; and another of $n=9$ items with mean $\bar{x}_2=+3$, and standard deviation $s_2=3/2$.

Then the standard deviation, s_d, of $d=\bar{x}_2-\bar{x}_1$ is

$$s_d=s\sqrt{(1/m)+(1/n)}=\frac{2}{45}\sqrt{255},$$

where
$$s^2=\frac{1}{m+n-2}[(m-1)s^2_1+(n-1)s^2_2]=32/15$$

$$t=d/s_d=2.8179.$$

Turning to the "Student" table in *Metron* with $t=+2.8179$, we find $p=.9935$ for the probability that t will fall within the range $-\infty$ and $+2.8179$ and $P=2\ (1-\mathrm{p})=.0130$ for the probability that t will fall outside the range -2.8179 to $+2.8179$. On this basis, t would fall outside the range about once per 77 cases.

Under the older procedure, taking the probable error of the difference, $\bar{x}_1-\bar{x}_2$, as the square root of the sum of the squares of the separate probable errors of $\bar{x}_1$ and $\bar{x}_2$, we should have $\bar{x}_1-\bar{x}_2=2$ with a probable error 0.48. Under this older method we should expect a difference as large or larger than 2 in only 31 cases out of 1,000,000, or in about 1 case per 33,000.

[1] Estimate of standard deviation of the population from the sample. Variance and standard deviation will be used in this sense throughout the remainder of the paper.

Consider next the following data on the yields of Indian corn on ten subdivisions of equal areas of two agricultural plots in which Plot 1 was a control plot treated the same as Plot 2, except for the amount of phosphorus applied as a fertilizer.

Yields in bushels on subdivisions of Plot 1	*Yields in bushels on subdivisions of Plot 2*
6.2	5.6
5.7	5.9
6.5	5.6
6.0	5.7
6.3	5.8
5.8	5.7
5.7	6.0
6.0	5.5
6.0	5.7
5.8	5.5
10)60.	10)57.0
$\bar{x}_1 = 6.00$	$\bar{x}_2 = 5.7$

On Plot 1, the variance is $s^2_1 = .64/9 = .0711$.
On Plot 2, the variance is $s^2_2 = .24/9 = .0267$.
The variance of $\bar{x}_1$ is $s^2_1/10 = .00711$ and that of $\bar{x}_2$ is $s^2_2/10 = .00267$.

$$\sqrt{s^2_1/10 + s^2_2/10} = .0989$$
$$.6745\sqrt{s^2_1/10 + s^2_2/10} = .066.$$

Under the older conventional treatment, assuming all distributions normal, the probable error for the difference .30, would be .066. In this older treatment, the inference would be that a difference as large or larger than .30 would be expected to occur in random sampling roughly about one time in 180,000 cases.

In application of the theory of small samples, we find

$$s^2 = \frac{1}{18}[.64 + .24] = .04889$$
$$s = .2211$$
$$s\sqrt{\frac{1}{m} + \frac{1}{n}} = s\sqrt{.2} = (.2211)\ (.4472) = .09888$$
$$t = (\bar{x}_1 - \bar{x}_2)/.09888 = .30/.09888 = 3.034.$$

From "Student's" tables in *Metron*, (*loc. cit.*) with $t = +3.034$, we find $p = .9964$ for the probability that t will fall into the range $-\infty$ to 3.034, and $P = 2(1-p) = .0072$ for the probability that t will fall outside the range -3.034 to $+3.034$. This should occur in the long

run about once per 139 trials which is very different from the once per 180,000 trials under the older normal theory. A simple comparison of corresponding probabilities of a discrepancy greater than an assigned multiple of the standard deviation as given by the "Student" table and the normal probability table, makes it obvious that the values in the "Student" table are larger, for small numbers in the sample, than the corresponding values in the normal table. The above illustrations are designed to assist in forming a rational belief as to the significance of these differences in relation to certain concrete applications. However, such beliefs should be dampened by emphasizing over and over the dangers of invalid assumptions about the randomness of our samples, and the normality of our parent distributions. The new procedure has probably been oversold by neglecting such emphasis in applications.

Before leaving the problem of testing the validity of the assumption that a sample belongs to a specified normal population, attention should be called to the fact that further methods than those discussed above have been developed. It seems very natural to anticipate that a useful test might depend on some other statistic or combination of statistics as well as on the ratio of mean of sample to standard deviation of sample. Neyman and E. S. Pearson have given within the past two or three years an illuminating analysis of the use and interpretation of various test criteria.[1]

It seems to be a fair inference from their analysis that four or five methods of making such tests are available to the statistician. The authors do not seem inclined to describe one test, on the whole, as sure to be more accurate than another, but should recommend one test or another according to the problem in hand. However, the "Student" procedure seems appropriate for a rather wide class of data drawn from a normal parent distribution.

While it would seem appropriate at this point to discuss the z-distributions of R. A. Fisher relating to the analysis of variance and correlation, this would require so much additional space that it seems better to limit our comments to testing the significance of means including non-normal universes than to attempt to cover more ground.

Since the exact theory of the procedure of testing the significance of differences of means of small samples inaugurated by "Student" and carried forward by R. A. Fisher is limited to cases in which the distribution of the parent population is normal, the question arises as to whether this procedure is to be preferred to the classical procedure or

[1] *Biometrika*, Vol. 20A, 1928, p. 175.

some other procedure in the case of non-normal parent distributions. This is a very important question for the reason that we do not ordinarily know whether a small sample is drawn from a normal population or from some other type of population. It is surely often drawn from some other type of distribution.

Some experimental work has been done bearing on the extent to which the procedures of "Student" and of R. A. Fisher apply when the sampled universe is not normal.

Shewhart [1] and Winters made an interesting contribution throwing some light on the question in 1928. They dealt experimentally with distributions obtained from 1,000 samples of four items drawn from three types of universes—normal, rectangular, and right triangular. It was found, for the sampling from the normal universe, that the "Student" procedure gave a good representation of the experimental distributions considering the fact that the range of the experimental normal universe is necessarily limited. For both a rectangular universe and a right triangular universe, the distribution of means and standard deviations of samples of four, conformed fairly well to the corresponding distributions from a normal universe, but the distribution of the ratio of mean to standard deviation $\bar{x}/s$ was so far from the "Student" values as to make it appear that the "Student" procedure is not applicable to such non-normal parent distributions as those used in this experiment. It was found that as the absolute value of $\bar{x}$ increases, the corresponding average values of s decrease very much. This result differs much from that of a normal parent distribution where $\bar{x}$ and s are independent. There were a larger number of z's deviating numerically an amount z_1 or more from expectation than is the case for the z's from a normal parent distribution. That is to say, in this experiment, the estimated probability obtained from the "Student" table that the ratio $\bar{x}/s$ will fall outside the interval $-z_1$ to $+z_1$ ($z_1 \leqq 3$) is too small for the sampling of four items from a rectangular or right triangular universe.

Professor Paul R. Rider [2] established this conclusion theoretically in 1929 for samples of two and four from rectangular universes of five and of ten class frequencies. The effect involved in this conclusion has been traced to the nature of the regression of the variances of the samples on the means of samples.

As a further experimental study, J. Neyman [3] and E. S. Pearson in 1928 made use of a set of sampling data obtained by Dr. A. E. R.

[1] This JOURNAL, No. 162, 1928, pp. 144–153.
[2] *Biometrika*, Vol. XXIA, 1929, pp. 124–143.
[3] *Biometrika*, Vol. 20A, pp. 197–201.

Church. The data were 1,000 samples of 10 drawn from a smooth distribution with the following frequency constants:

$$\sigma = 3.569{,}207$$
$$\beta_1 = .219{,}333$$
$$\beta_2 = 3.157{,}676.$$

This distribution not differing much from a Pearson Type III form, may be taken as typical of moderately skew distributions often met in practice. Church found that the distribution of means is well described by a normal distribution. For the distribution of s, there appeared to be, on the whole, too few low and too many high values of s among the observations.

While the differences between the observed distribution of $z = \bar{x}/s$ and the corresponding distribution of z when samples are drawn from a normal parent distribution are not very pronounced, there is no doubt that there are among the observations too few with large negative values and too many with large positive values of z to conform well to the case when the parent distribution is normal. The observed distribution of z is significantly skew, whereas "Student's" distribution is symmetrical. It was inferred by Neyman and Pearson that they were probably just reaching a degree of skewness in the population for which the theory for a normal parent distribution begins to fail.

E. S. Pearson and N. K. Adyanthāya published in *Biometrika* for December, 1929, a further experimental study of the distribution of the numerical values of z from a variety of non-normal parent distributions for samples of 2, 5, 10 and 20 items. Out of this study there has come some evidence suggesting a practical procedure for passing judgment on the significance of the value of z when the parent distribution is non-normal.

While no entirely satisfactory solution of the problem seems possible unless some one succeeds in deriving the theoretical distribution of z for the non-normal population with which we are concerned, it seems that it will facilitate applications, in the present state of knowledge, to deal with the probability that the value of z will lie within or without an assigned interval $-z_1$ to $+z_1$ from expectation rather than to deal with the probability that z will lie in the interval $-\infty$ and z_1. This procedure makes use of the balancing of the defects of frequencies of z's in the interval $-\infty$ to $-z_1$ against an excess in the interval z_1 to ∞, and vice versa. Pearson and Adyanthāya used this fact in the analysis of their experimental distributions of z. They found that great caution must be exercised in drawing conclusions from apparent differences in their cumulative frequency distributions.

The evidence brought forward by the results of the experiment suggests that, until there is further evidence, the practical procedure to adopt is to make use of the probabilities derived from the "Student" tables, that the numerical value of z is less than or exceeds an assigned value. By using this procedure for distributions with only a moderate degree of skewness and peakedness, there will probably not be very serious danger, in actual applications, of calling differences significant when they are insignificant.

In the light of the theoretical and experimental results on small samples or perhaps it would be better to say, in the present state of our knowledge, it appears that we stand in need of further investigation of the applicability of the "Student" procedure to samples from universes differing widely from normal but that in so far as we have evidence, the "Student" procedure would, ordinarily, seem much more likely to be appropriate in such cases than the older procedure.

In the foregoing comments, an attempt has been made to emphasize the limitations on the theory of small samples involved in the assumption that the parent distribution is normal; for when we consider the effect of this limitation, it is fairly obvious that the problem of small samples is not solved in the exact general form in which it may be made to appear in statistical practice. While one limitation is thus emphasized, lest we forget, it seems well to emphasize in conclusion that the validity of the applications are dependent on drawing a random sample. This condition is likely to be relatively even more difficult to satisfy in statistical practice with small samples than with large samples. This condition is a factor in making it desirable to avoid small samples when possible, but it should hardly prevent the drawing of probable inferences from small samples when it is impracticable to obtain more data.

In conclusion it may be of interest to some readers that Professor Paul R. Rider [1] has given a most excellent "Survey of the Theory of Small Samples" in the October, 1930, number of the *Annals of Mathematics*. He has put much into small space, and has provided very complete references to the memoirs in which the theory is developed.

[1] See also his summary, this issue of this JOURNAL, p. 172.

THE STABILIZED NATURAL INCREASE OF THE NEGRO

By S. J. Holmes and S. L. Parker

Until recently little attention has been paid to what may be called the stabilized rate of population growth, or the rate at which the population would increase after it had outgrown the anomalies of its age composition. It is well known that the surplus of births over deaths in a given region is to a large extent dependent upon the age distribution of the inhabitants. The age composition of the population of the United States is exceptionally favorable for a rapid natural increase because of the high proportion of persons in the reproductive period of life. This is due partly to foreign immigration and partly to the decline in the birth rate, which reduces the proportion of children. Naturally, a net rate of increase due to such causes cannot be expected to continue. If the fertility and mortality rates were to remain constant in each age group of the population, birth rates would soon come to fall and death rates to rise as the population approaches a stabilized condition, as it would inevitably tend to do.

The importance of calculating stabilized rates has been emphasized especially in the recent publications of Dublin and Lotka and Kuczynski. In a paper by Dublin and Lotka on "The True Rate of Natural Increase" [1] it is shown that, when allowance is made for the anomalies of age composition, the death rate of the white population of the Birth Registration Area in 1920 would be 15.4 per 1,000, the birth rate 20.9 per 1,000, and the net increase about 5.5 per 1,000, or approximately half the rate obtained by simply subtracting the crude death rate from the crude birth rate. In a later paper [2] the authors have made an estimate of what the "true," or stabilized natural increase would be in the years from 1920 to 1928. By 1928 the true rate of natural increase was estimated to have fallen as low as 1.7 per 1,000. With our restricted immigration we shall be, apparently, in no serious danger of overpopulation for several years to come.

From his studies of the stabilized natural increase of the peoples of Northern and Western Europe, Kuczynski [3] has found that, despite a very considerable surplus of births over deaths, the present reproductive rate of this region as a whole, as well as that of most of the

[1] L. I. Dublin and A. J. Lotka, "On the True Rate of Natural Increase," this Journal, 1925, pp. 305–339.

[2] L. I. Dublin and A. J. Lotka, "The True Rate of Natural Increase of the Population of the United States," *Metron*, Vol. 8, 1930, pp. 107–119.

[3] R. R. Kuczynski, *The Balance of Births and Deaths*, New York, Macmillan, 1928, pp. xi, 140.

component countries, would not suffice to maintain the population were it not for the favorable age distribution of the inhabitants.

Owing to the marked differences which may occur between crude and stabilized rates of natural increase, it was considered a matter of interest to obtain at least an approximate estimate of the stabilized increase of our Negro population. After this work had been nearly completed, we became aware of the recently published papers[1] of Professor Whelpton, which contain estimates of the stabilized increase of the Negro as well as the white population of certain sections of the United States. The areas dealt with by Whelpton are somewhat different from those studied by us, and the treatment of the Negro population forms a rather minor part of the papers referred to, but the general conclusions arrived at are very similar to those which we had reached. The work of Professor Whelpton is based upon certain assumptions as to the Negro birth rate, which will be discussed later. On account of the uncertainty as to the deficiencies of birth registration among Negroes, it was deemed of interest to calculate stabilized rates of increase on the basis of a number of different assumptions as to the birth rate.

According to the vital statistics of the Registration Area for Births in 1920, the birth rate of the Negro population was 26.3 per 1,000, and the death rate 18.4 per 1,000, leaving a net gain of 7.9 per 1,000. At the same time the corresponding figures for the whites in the same area were 23.5 per 1,000 and 12.8 per 1,000, leaving a net increase of 10.7 per 1,000. Hence the whites in this region were increasing at a considerably more rapid rate than the Negroes.

Unlike the whites, the Negroes of the United States owe relatively little of their increase to immigration from abroad; and the number derived in this way is partly offset by emigration to Canada and other countries. Whatever changes have taken place in the age composition of our Negro population as a whole have, therefore, been due mainly to changes in the birth rate and the death rate.

It is, however, impossible to calculate the stabilized natural increase of our entire Negro population because some of the states of the South are still outside the Registration Area for Births, and adequate data on Negro mortality are not available for the United States as a whole. Our calculations are, therefore, confined to the Registration Area for Births and to the year 1920, because data on age composition are available only for that year. It is not considered safe to make estimates of the stabilized rate of increase subsequent to 1920 on account of the uncertain extent of interstate migration and

[1] P. K. Whelpton, "Differentials in True Natural Increase," this JOURNAL, 1929, pp. 233–249.

other factors which would lead to errors in estimates of total population, age grouping, and the proportion of sexes, to say nothing of the relation of births to deaths.

The procedure followed was that of ascertaining how many daughters would be produced to every 100,000 Negro females born in the area investigated. For this purpose one needs to know the number of survivors living in the several age groups for every 100,000 females born, and how many children are produced by the surviving females of each age group. Data on the first point may be obtained from the life tables for 1920, and on the second from the volume on birth statistics for the same year.

The *United States Abridged Life Tables*[1] for 1919–1920 give the mortality of Negro females in a number of areas. Unfortunately, none of the mortality tables applies to the whole Registration Area for Births. The table for states having over 5 per cent of Negro population includes two states, Tennessee and Missouri, for which data are not available for births in the several age groups. Unless Negro mortality in these two states differed markedly from that of the rest of the area, the procedure followed would not greatly affect the estimate of the average mortality of the entire group. No great error, therefore, would be introduced by applying the mortality rates of the seven states in question to the five states (Kentucky, Maryland, North Carolina, South Carolina and Virginia) for which we have data on the birth rates of females in the several age groups.

Table I gives the data on which the estimate of population increase

TABLE I

THE STABILIZED INCREASE* OF NEGROES IN FIVE SOUTHERN STATES

1 Age	2 Number of births	3 Births per 100,000 females	4 Survival factor	5 3×4
10–15	152	84.93	.84559	71.81
15–20	6,784	4,286.48	.81958	3,513.11
20–25	12,490	8,406.76	.77279	6,496.66
25–30	8,665	7,527.52	.71932	5,414.70
30–35	5,270	6,108.37	.66488	4,061.33
35–40	4,220	4,490.13	.61232	2,749.40
40–45	1,288	1,820.55	.56193	1,023.02
45–50	206	354.06	.50954	180.41
50–55	10	22.37	.45388	10.15
Total	39,085	33,101.17 × 5		23,520.59 × 5
Daughters per 100,000 females		165,505.85	Stabilized rate per 100,000 females	117,602.95

$$\text{Annual rate} = \sqrt[27.5]{1.17603} - 1$$

$$= .0059, \text{ or } 5.9 \text{ per } 1{,}000$$

*For further information on the method of computation see Dublin and Lotka "On the True Rate of Natural Increase."

[1] E. Foudray, *United States Abridged Life Tables*, Washington, D. C., 1928, p. 84.

is based.[1] In the second column are the numbers of daughters born to mothers in the age groups indicated in column 1. The third column gives the number of daughters produced by 100,000 women in the several age groups. Column 4 gives the ratio of the number of women surviving in the several age groups out of every 100,000 born. For instance, the first number of this column states that out of 100,000 females born there are 84,559 surviving who have reached the mid-age of the 10 to 15 age group. The items of the last column, 5, are obtained by multiplying together those of columns 3 and 4. The sums obtained by adding the entries in columns 3 and 5 are multiplied by 5 because each age group covers a period of 5 years. The total of column 3 multiplied by 5 gives the number of daughters (165,506) who would be produced if all mothers survived to age 55. The total of column 5 multiplied by 5, namely, 117,603, gives the total number of daughters who would be produced by 100,000 females after making allowance for the number who died at various ages before the completion of the child-bearing period. It would therefore represent the actual fertility of a stabilized population. If this number exceeds 100,000, the population would increase; if it falls below, there would be a decrease. Since the number considerably exceeds 100,000, it is evident that the Negro population of the five states in question would be increasing at a fair rate. Every 100,000 Negro females born would give rise to 117,603 females to replace them in the next generation. Actually, 165,506 females are produced for every 100,000 mothers, but a considerable number of these daughters, of course, fail to attain adult age. The stabilized rate of increase among the Negroes in the five southern states is slightly more rapid than that of the whites in the total Registration Area for Births.

In order to ascertain how Negroes promise to thrive in the North, a similar estimate was made for several of the northern states in the Registration Area for Births in 1920, namely, California, Connecticut, Indiana, Kansas, Massachusetts, Michigan, Minnesota, New York, Ohio, Oregon, Pennsylvania, Utah, Washington and Wisconsin. From the volume on Birth Statistics for 1920 we obtained the number of female children born to Negro mothers in the various age groups, and the number of Negro females in these same age groups. In estimating the survivors in these age groups out of 100,000 females born, we employed the data in the *United States Abridged Life Tables* for 1919–1920, but, as in the estimate of the increase of the five southern states, we had to apply death rates based on a slightly different territory.

[1] Children born to mothers of unspecified age were distributed among the various age groups in proportion to the size of the respective groups.

The life tables give data on the Negro mortality for a group of states having less than 4 per cent of Negro population. This group is the same as that we have just mentioned, except that it includes Illinois and New Jersey. Naturally, a source of error is introduced by this procedure, but it cannot be large.

The calculation shows that in 1920 in these northern states, 100,000 Negro females at birth would, at the prevailing rate of increase, produce only 73,213 females to replace them. This is a very striking fact. It

TABLE II

THE STABILIZED INCREASE OF NEGROES IN FOURTEEN NORTHERN STATES

1 Age	2 Number of births	3 Births per 100,000 females	4 Survival factor	5 3×4
10–15	34	93.31	.78799	73.53
15–20	1,762	4,603.53	.75907	3,494.40
20–25	3,502	6,155.31	.71670	4,411.51
25–30	2,685	4,445.07	.67586	3,004.25
30–35	1,522	3,118.92	.63664	1,985.63
35–40	957	1,953.58	.59585	1,164.04
40–45	279	806.87	.55221	445.56
45–50	31	116.86	.50378	58.87
50–55	2	10.63	.44773	4.76
Total	10,744	21,304.08 × 5		14,642.55 × 5
Daughters per 100,000 females		106,520.40	Stabilized rate per 100,000 females	73,212.75

$$\text{Annual rate} = \sqrt[27.5]{.73212} - 1$$

$$= -0.0113, \text{ or } -11.3 \text{ per } 1{,}000$$

means that in 1920 the North was highly unfavorable for the natural increase of the Negro race, because, under the conditions prevailing at that time, the Negroes in the North would be rapidly decreasing in number.

The preceding calculations, like those of Dublin and Lotka, were made upon the assumption that all births were registered in the areas covered. This assumption, of course, cannot be true. Presumably, birth registration is at least 90 per cent complete if a state is admitted to the Registration Area for Births. From time to time the accuracy of birth registration is tested. In 1916 the Bureau of the Census made an investigation of the completeness of birth registration in New England by ascertaining how many deaths occurred in children under one year of age for which no birth certificates could be found. The percentage varied in different states from 5.6 to 18.3 per cent. Further investigation, however, showed that birth registration was more than 90 per cent complete in the two states, Connecticut and Rhode Island, which apparently did not come up to the standard, although Rhode

Island was removed from the Birth Registration Area in 1919 "because of failure to send in all transcripts of birth certificates for the year."

An investigation of the adequacy of birth registration which was carried on in connection with the compilation of the United States Life Tables for 1919–1920 showed that "the number of births registered ranged from about 75 per cent to possibly 100 per cent of the actual number of births." Instead of using the number of births given in the reports on birth statistics, the number was calculated on the basis of the census returns for young children, the data on infant mortality, and other information which could be gathered in regard to unregistered births in a sample area. In order to obtain an estimate of how great an allowance should be made for the shortcomings of birth records, a canvass was made of both the white and the Negro population of the District of Columbia.

> It was then assumed that the per cent of infants whose names were missing from the census schedules for the District was constant throughout the United States, and the per cent determined was used to obtain a close approximation to the actual number of children under one year of age in each community on January 1, 1920. Different constants were determined for white and Negro children. The constant per cent of children whose names were missing was taken as 9 for whites, and 25 for Negroes.

With this assumption the number of births among whites and Negroes was estimated on the basis of the population and infant mortality statistics of the several states and cities. The "adjusted" number of births was found to be higher than the registered number by a percentage which varied in different regions. In the states with less than 4 per cent of Negroes it was estimated that in 1920 there were 17,027 female births, while the number given in the birth statistics was 13,563, an increase of 25.54 per cent. For states with more than 5 per cent of Negroes the adjusted number of female births was estimated to be 59,378, while the reported number was 43,844, an increase of 35.43 per cent.

In Professor Whelpton's estimates of the stabilized rates of increase, use was made of the adjusted number of births given in the 1919–1920 Life Tables. This procedure naturally makes the stabilized rates of increase of the Negroes considerably greater than those which we have estimated on the assumption of complete registration of births. For seven southern states he obtains a stabilized rate of 13.0 per 1,000, and for sixteen northern states a rate of −4.8 per 1,000. Our estimates give a rate for five southern states of 5.9 per 1,000, and for fourteen northern states a rate of −11.3 per 1,000.[1]

[1] The average length of a generation among the northern group of Negroes is 27.65 years, and for the southern group 27.43 years. Since a difference of a half year in the average length of a generation

Unfortunately, we have at present no way of estimating the rather large element of uncertainty as to the actual birth rates of the Negroes. Year after year our volumes on birth statistics repeat the statement that "it is possible that registration of births is not as complete among colored as among white persons, and that, therefore, rates shown for the former class are too low." That the Negro birth rate in Washington, D. C., was considerably higher than is shown by the birth statistics seems evident from the investigation to which we have referred. The discovery that some 25 per cent of Negroes escaped enumeration in the 1920 Census is somewhat disconcerting. The additional names used in the investigation made in compiling the United States Life Tables were secured from birth records and infant mortality records.

> A form letter was sent to the parents of all those children whose names did not appear either in the census schedules of January 1, 1920, or on the death records for the District for 1919. Between 500 and 600 answers to these inquiries were received, and they were used as a basis for estimating the status on January 1, 1920, of the children whose names were missing from the schedules and about whom it was impossible to obtain definite information.

How many of these replies came from Negro families we do not know, nor is any statement made as to the proportion of families from which no replies were received. It was apparently assumed in estimating the number of births in the District of Columbia in 1919 that all children under one year of age enumerated in the census must have been born in the District. Owing to the large influx of Negroes into Washington, it is not unlikely that many infants were brought in during 1919 who were born elsewhere and whose birth was thus credited to the District. A method of estimating the birth rate which would give reliable results for an entire country is liable to several sources of error when applied to a relatively small district whose population has recently received considerable additions by migration. No statement is made in the United States Life Tables as to any attempt to eliminate this source of error. Even if such an effort were made, carelessness on the part of parents or census enumerators might lead to births being attributed to Washington which did not occur there.

It is commonly assumed that birth statistics always fall short of giving the full number of births. An investigation carried out by the United States Children's Bureau in a limited district of Massachusetts, a state with unusually complete registration of births, revealed that "99 births were found to have been registered twice, 10 births were

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makes only a very slight difference in the stabilized rate of increase, we have assumed the length of a generation in both cases to be 27.5 years, which is about one year less than the average length of a generation among the whites.

registered which actually occurred outside the limits of the municipality, 10 births occurred in another year from that in which they were registered." One hundred twenty-three births were not registered. For one reason or another the errors happened to offset one another fairly well, so that the reported number was only 14 less than the number finally ascertained. Doubtless, these various sources of error have very different effects in different places. The results throw some doubt upon the adequacy of checking up birth registrations by simply finding the number of unregistered births in a given community.

On the whole, the effect of urban migration is probably to cause estimates of the birth rate by the method described to be too high. How much too high they may be we have no good means of ascertaining. There is reason to believe that the adequacy of birth registration among the Negroes, as among the whites, varies greatly from place to place, and adjustments made by applying results derived from a study of the District of Columbia to other localities may lead to conclusions which are far from correct. We really do not know whether birth registration among the Negroes falls short of completeness by 10, 15, 20 or even 25 per cent. By making "adjustments" on the basis described in the United States Life Tables, the shortage may be even greater. The true stabilized rates of the Negro probably lie somewhere between those we have obtained on the assumption of complete registration and the results obtained by Professor Whelpton. In view of the uncertainties involved, we have drawn no conclusions as to what the true birth rate of the Negro actually is. It was considered to be more useful to calculate the stabilized increase of the Negroes on the basis of a number of different assumptions as to the completeness of birth registration. Accordingly, in addition to calculating the stabilized increase as it would be if birth registration were complete, we have estimated what it would be if birth registrations fell short of completeness by 10, 15, 20 and 25 per cent. Under these conditions the stabilized rates for northern Negroes would be -7.6, -5.5, -3.3 and -0.9, respectively, and the stabilized rates for southern Negroes would be 9.7, 11.8, 14.0 and 16.4.

We have compiled a general table (Table III) showing how different stabilized rates of increase would be affected by various assumptions as to the completeness of birth registration. The first column gives the assumed stabilized rates per 1,000 of the population on the assumption that birth registration is complete, and the other columns indicate the rates as they would be if birth registration were incomplete to the extent indicated by the percentages given at the heads of the columns. Thus, a stabilized rate of -13 per 1,000 would be increased to -9.6 per 1,000

TABLE III

STABILIZED RATES OF INCREASE AS AFFECTED BY DIFFERENT BIRTH RATES

Rates, computed on assumption of complete registration	Rates computed on assumption of incomplete registration to extent indicated by percentages 10–35					
	10 per cent	15 per cent	20 per cent	25 per cent	30 per cent	35 per cent
−13.0	−9.6	−8.0	−6.4	−5.0	−3.5	−2.1
−11.0	−7.6	−6.0	−4.4	−2.9	−1.5	−0.1
− 9.0	−5.6	−3.9	−2.4	−0.9	−0.5	2.0
− 7.0	−3.6	−1.9	−0.4	1.1	2.5	4.0
− 5.0	−1.5	0.1	1.6	3.0	4.6	6.0
− 3.0	0.5	2.1	3.6	5.1	6.6	8.0
− 1.0	2.5	4.1	5.6	7.1	8.6	10.0
1.0	4.5	6.1	7.7	9.2	10.6	12.0
3.0	6.5	8.1	9.7	11.2	12.6	14.0
5.0	8.5	10.1	11.7	13.2	14.6	16.1
7.0	10.5	12.1	13.7	15.2	16.7	18.1
9.0	12.5	14.1	15.7	17.2	18.7	20.1
11.0	14.5	16.2	17.7	19.2	20.7	22.1
13.0	16.5	18.2	19.7	21.3	22.7	24.1

if birth registration were assumed to be 10 per cent incomplete, or to −2.1 per 1,000 if it fell short of completeness by 35 per cent. The differences between the first and the following columns are nearly constant, being from 3.4 to 3.5 for an assumed incompleteness of 10 per cent in birth registration, and from 10.9 to 11.1 for an assumed incompleteness of 35 per cent. In order to see how an assumed inadequacy of birth registration of 25 per cent would affect a stabilized rate of 10 per 1,000, one can add the corresponding difference, 8.2, and obtain the rate of 18.2. These numbers are calculated on the assumption of 27.5 years to a generation. An increase or a decrease of a half year would affect these numbers but slightly. A generation of 28 years would reduce the numbers to be added for a 10 per cent incompleteness of birth registration from 3.5 to 3.4, and for a shortage of 35 per cent, from 11.0 to 10.8.

Whichever figure is taken for the stabilized increase in the five states of the South, if the same figure obtains for other southern states as well, the gains in the South would more than counterbalance the losses in the North, owing to the much larger Negro population in the southern states. Up to 1920 deaths among Negroes have exceeded births in most states of the North, and in 1920 only four states, New York, Pennsylvania, Massachusetts and Connecticut, showed more births than deaths in the Negro population, and this condition had obtained in these four states for only a short time. Nevertheless, the excess of births in these states was sufficient to give a slight preponderance of Negro births, namely 412, in the northern states of the Birth Registration Area of 1920. Northward migration has proven a terrific drain upon the Negro population of the United States. The Negroes have gone mainly into the cities of the North, where they were exposed to a

colder climate under living conditions which were commonly far from wholesome. The varied circumstances which lead to a restriction of the birth rate in urban populations have had a marked effect upon the fecundity of the northern Negro. That the stabilized rate of increase of the northern Negro was so low in 1920, despite the slight surplus of births over deaths, is due largely to the circumstance that the Negro population contained relatively few old people and young children. The age composition of northern Negroes was therefore especially favorable for a high birth rate; but the Negroes failed to reproduce as rapidly as might be expected. To a considerable extent, the slow multiplication of the northern Negroes is due to their high death rate. Mortality in infancy and childhood has been high, and the death rate from tuberculosis among adolescent Negroes has been several times that of the corresponding age groups among the whites. Another important factor is that a considerable proportion of northern Negroes in 1920 were recent migrants from the South who came during or soon after the War, and had not become well established. Although many young women came North, so that the proportion of sexes was not particularly anomalous in the North as a whole, the men and women often went into different areas, as is indicated by the very different sex ratios in the colored population of several northern cities.

The wholesale migration of Negroes into northern cities brought many trying difficulties of adjustment. It often meant the separation of the wage earners from the rest of the family, the postponement of marriage, and a strong economic temptation to limit the birth supply. In addition to these and other causes which would tend to reduce the actual rate of natural increase, there are some purely statistical factors which should not be neglected.

Let us consider the influence of migration on the balance of births and deaths. The transportation of infants and young children from South to North during the period covered by our statistical data credits the North with the deaths of many persons without giving it credit for their births. The mortality in the North is also increased by the migration of older Negroes. Ordinarily, the migration of Negroes in adolescent and middle age might be expected to increase the birth rate in the North, since there would be more births than deaths in this group, as there have been among our foreign immigrants. But this group, as we have explained, doubtless has been rendered relatively sterile through being transplanted into a new environment to which it has made only a very imperfect adjustment. The South was left with a Negro population less favorable for rapid multiplication, and hence there would be a tendency for the death rate to rise and for the birth

rate to fall. The increase of northern Negro migrants was reduced far below what it probably would have been had these migrants remained in the South.

The effect of the dislocation has therefore been to reduce the total increase of our colored population. This effect may, however, be only temporary. The void left in the South will gradually be filled up as the younger Negroes pass into the reproductive period. The Northern migrants, after their period of adjustment, have shown evidence of responding by an increased rate of multiplication. If we may rely upon the vital statistics published since 1920, there has been a general, and in some cases a striking, rise of the Negro birth rate in the North during the past decade. At the same time, there has been a marked reduction in Negro infant mortality. The infant mortality rate in the northern states of the Birth Registration Area was 160.5 in 1920, and 105.6 in the same states in 1927.

In 1927 births had come to exceed deaths among the Negroes in Connecticut, Delaware, Illinois, Indiana, Massachusetts, Michigan, New Jersey, New York, Ohio and Pennsylvania. Taking six representative northern states of the Birth Registration Area in 1920 (New York, Pennsylvania, Ohio, Indiana, Michigan and Massachusetts) we find that there were 19,219 births and 18,337 deaths, leaving a surplus of 882 births over deaths. In 1927 there were 31,371 births and 24,226 deaths, giving a surplus of 7,145 births over deaths. At the latter date (1927) there were 12,152 more births and 5,889 more deaths than in 1920. These states have received a large influx of Negroes, and the recent trend of their vital statistics indicates that the natural increase of their Negro population is considerably more rapid than it was in 1920. Whether the age composition of the Negro population of these states has been rendered more favorable for rapid multiplication than it was in 1920 we do not know. It seems much more probable that the statistical increase of the northern Negroes represents a true increase in their rate of multiplication. This increase has been so marked and so general that one cannot be certain that were it possible to obtain an accurate measure of the stabilized reproductive rate at the present time it might not show a surplus of births over deaths.

Even if a stabilized reproductive rate of the northern Negroes is not sufficient to insure their perpetuation, it probably comes more nearly doing so than it did in 1920. As one of the writers pointed out in a paper published about two years ago,[1] the northern Negroes are actually more than holding their own. In this paper it was stated that

[1] S. J. Holmes, "Will the Negro Survive in the North?" *Scientific Monthly*, Vol. 27, 1928, pp. 557–561.

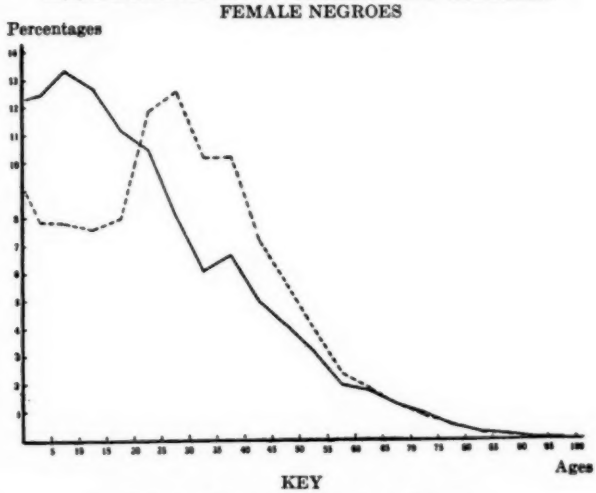
this fact might be due to the favorable age composition of the northern Negroes and hence would not be permanent. The conclusion was expressed that "while it may be too soon to assert that the northern Negroes will continue to increase through their own birth rate, the conclusion that they cannot survive in a higher latitude is hardly warranted by existing facts." The calculation of stabilized rates of reproduction has shown that the actual surplus of births over deaths among northern Negroes would not have occurred, in 1920 at least, were it not for their unusually favorable age composition. Professor Whelpton has made an estimate of the stabilized reproductive rate in 1928 and concludes that it is considerably greater than in 1920. While there are several sources of error in such an estimate, the evidence, so far as it goes, corroborates the conclusion to which we have arrived. According to Whelpton,[1] the stabilized reproductive rate of northern Negroes changed from −4.8 per 1,000 in 1920 to −1.1 per 1,000 in 1928. The actual decrease would probably be somewhat more rapid than is represented by the latter figure, but the trend of their vital statistics indicates that the northern Negroes are approaching a stabilized reproductive rate which would yield an actual increase in numbers. This trend, however, may be reversed by the decline of the birth rate during the next few years.

In order to illustrate the peculiarities of age composition, upon which such marked differences between true and stabilized rates of increase may depend, the chart has been prepared showing the age distribution of northern and southern Negroes. The same five southern states and fourteen northern states have been employed in calculating stabilized rates of increase. The chart indicates the age distribution of females as given in the 1920 Census. An inspection will reveal at once the marked difference in age composition between northern and southern Negroes. In the light of the fact here illustrated it is not surprising that there should be a very striking difference between the actual and the stabilized increase in these groups, especially the Negroes of the North.

For the sake of comparison, an estimate was made of the stabilized natural increase of the white population of the North and the South. We have employed the same groups of states that were used in calculating the stabilized increase of the Negroes. In the absence of life tables covering these groups of states, the rates given for the female white population in the United States Life Tables of 1919–1920 were used. There are obvious sources of error in this procedure because the mortality of women in the several age groups is not the same in all areas. The results, therefore, are only crude approximations. It, however, may

[1] P. K. Whelpton, "Population. Trends in Differentials of True Increase and Age Composition," *American Journal of Sociology*, Vol. 35, 1930, pp. 870–880.

AGE DISTRIBUTION OF NORTHERN AND SOUTHERN FEMALE NEGROES



KEY

Unbroken line represents southern Negroes.
Broken line represents northern Negroes.

be of some interest to note that the stabilized increase of white women in five southern states estimated in this crude way is 145,400, while that of white women in the states of the North is 111,707. In both areas the stabilized increase of white women in 1920 was much greater than that of Negro women. This fact is all the more striking when we consider that the crude birth rate for the Registration Area as a whole in 1920 was 26.3 for the Negroes, and 23.5 for the whites. At the same time, the Negro women of the South were reproducing more rapidly than the white women of the North. They were reproducing a little more rapidly than the white women of the total Registration Area, their rate being such that for every 100,000 females born 115,794 daughters would be produced in the next generation. It should be borne in mind that the Negro population of the five southern states (2,798,560) was nearly three times that of the fourteen northern states (997,813), and that the white population of the northern states was over six times that of the five states in the South. Were it possible to include the total population of both races it is probable that the stabilized rates of the Negroes would be relatively higher in relation to the total white population. The Negroes left out of the reckoning are those constituting the mainly rural population of Georgia, Alabama, Mississippi, Louisiana, Texas, etc., where the available evidence is indicative of a high birth rate. It is the rural South which, up to 1920 at least, has prevented the actual decrease of our Negro population.[1]

[1] S. J. Holmes, "The Biological Trend of the Negro," *University of California Chronicle*, Vol. 31, 1930, pp. 38–70; W. S. Thompson, *Population Problems*, New York, McGraw-Hill, 1930, pp. xi, 462.

NOTES

A NOTE ON SMALL SAMPLE THEORY [1]

By Paul R. Rider, *Washington University*

It may be of interest in connection with Professor Rietz's [2] paper to sketch briefly the way in which R. A. Fisher has unified the whole theory of small samples from normal populations.

After placing the derivation of "Student's" distribution of the ratio of mean to standard deviation on a rigorous basis, Fisher [3] showed that this distribution has a much wider range of application than the problem for which it was originally designed. In particular he showed that the "Student" distribution can be used not only to determine the reliability of a mean, but also to test the significance of the difference between two means—Professor Rietz has illustrated both of these uses—and to test hypotheses regarding regression coefficients, linear, nonlinear, and multiple.

He followed up "Student's" paper on the "Probable Error of a Correlation Coefficient," [4] and determined the exact distribution of the simple or interclass correlation coefficient in random samples.[5] He obtained also the distribution of the intraclass correlation coefficient.[6]

These various distributions, except that of the simple correlation coefficient, have been shown by Fisher [7] to be special cases of what he calls the z-distribution or the distribution of the analysis of variance, a term which he employs to indicate the resolution of the variance [8] into

[1] Revision of a paper delivered at the Ninety-Second Annual Meeting of the American Statistical Association at Cleveland, Ohio, December 30, 1930.

[2] See p. 150 this issue.

[3] R. A. Fisher, "Applications of 'Student's' Distribution," *Metron*, Vol. 5, No. 3, December 1, 1925, pp. 109–120.

[4] *Biometrika*, Vol. 6, 1908–09, pp. 302–310.

[5] "Frequency Distribution of the Values of the Correlation Coefficient in Samples from an Indefinitely Large Population," *Biometrika*, Vol. 10, 1914–15, pp. 505–521.

[6] "On the 'Probable Error' of a Coefficient of Correlation Deduced from a Small Sample," *Metron*, Vol. 1, No. 4, September 1, 1921, pp. 3–32.

An *intraclass* correlation coefficient is one in which it is assumed that the variates have a common mean and a common standard deviation. This might occur, for example, if one were endeavoring to find, for a certain characteristic such as stature, the correlation between pairs of brothers. (See Fisher's paper just referred to, or his book *Statistical Methods for Research Workers*.)

[7] See "On a Distribution Yielding the Error Functions of Several Well-Known Statistics," *Proceedings of the International Mathematical Congress*, Toronto, 1924, Vol. 2, pp. 805–813.

[8] The variance is the square of the standard deviation.

the portions contributed by two or more different causes. The quantity z is defined by the equation

$$z = \tfrac{1}{2} \log (s^2_1/s^2_2) = \log (s_1/s_2),$$

in which s^2_1 and s^2_2 are two independent estimates of the variance of a variate x which is normally distributed with variance σ^2. If the estimates s^2_1 and s^2_2 are based upon n_1 and n_2 "degrees of freedom" (i.e. upon samples of n_1+1 and n_2+1) respectively, then

$$s^2_1 = \frac{1}{n_1} \sum_{i=1}^{n_1+1} (x_i - \bar{x})^2, \qquad s^2_2 = \frac{1}{n_1} \sum_{i=1}^{n_2+1} (x'_i - \bar{x}')^2,$$

$\bar{x}$ and $\bar{x}'$ being the means of the two samples. It can then be shown that the probability distribution of z is

$$f(z) = \frac{2 n_1^{n_1/2} n_2^{n_2/2}}{B(n_1/2, n_2/2)} \frac{e^{n_1 z}}{(n_1 e^{2z} + n_2)^{(n_1+n_2)/2}}, \tag{1}$$

in which B is the beta function. It is most noteworthy that this distribution is absolutely independent of the variance of the sampled population. If s^2_1 and s^2_2 are estimates of two different variances, σ_1^2 and σ_2^2, we have merely to write $\zeta = \frac{1}{2} \log (\sigma_1^2/\sigma_2^2)$ and the distribution of $z-\zeta$ will be exactly the same as that of z in (1).

The special cases of the z-distribution that are of particular interest and importance are

1. $n_1 = n$, $n_2 = \infty$. By making the substitution $z = \frac{1}{2} \log (\chi^2/n)$ we are led to the Chi-square distribution employed for instance in testing the goodness of fit of observed data to theoretical frequency curves.

2. $n_1 = 1$, $n_2 = n$. The substitution $z = \log t$ leads to "Student's" t-distribution discussed by Professor Rietz.

3. $n_1 = 1$, $n_2 = \infty$. This yields the normal distribution if we set $z = \log x$.

4. $n_1 = n-1$, $n_2 = n$. If r is an intraclass correlation coefficient calculated from a sample of size n, then the substitution $z = \tanh^{-1} r = \frac{1}{2} \log [(1+r)/(1-r)]$ will give the distribution of r. Now the distribution of r is far from normal for small samples, and even for large samples when the correlation in the universe is high, and the advantages of using z instead of r are that the z-distribution is constant in form, being nearly normal, and that the standard deviation of z is independent of the correlation in the sampled population. Essentially the same advantages accrue if we use z in place of the ordinary or interclass correlation coefficient.

From the z-distribution can also be derived, for the case of samples from uncorrelated material, the distribution of the coefficient of multiple correlation and that of the correlation ratio.

With the discovery of the distribution of the coefficient of multiple correlation from correlated material,[1] the set of distributions of the statistical parameters in common use may be said to be complete. This distribution connects with that of the simple correlation coefficient and with that of the multiple coefficient from uncorrelated material. Perhaps the most interesting feature of the distribution is that it does not depend upon the whole matrix of correlations among the independent variates, but upon the multiple correlation in the parent population.

It must be remembered, however, as Professor Rietz has pointed out, that Fisher's results are all based upon the assumption of normality in the sampled population, and most of the present activity in small sample theory seems to center upon ascertaining to what extent the methods of "Student" and Fisher may be applied to samples from non-normal universes, and devising new methods where these are not applicable.

[1] R. A. Fisher, "The General Sampling Distribution of the Multiple Correlation Coefficient," *Proceedings of the Royal Society of London*, Series A, Vol. 121, 1928, pp. 654–673.

HAS ENGEL'S LAW ITS LIMITATIONS?

By Povl Heiberg, *Deputy City Medical Officer of Health of Copenhagen*

In the course of a study of some budget tables [1] based on the Danish family budget investigations of 1922, I was surprised to find that Engel's well-known law did not apply to the families of two members in Copenhagen included in the income groups (from 3,000 to 8,000 Kroner) investigated. Engel's law states that the proportionate expenditure on food decreases as the income increases. The law that he arrived at by induction is, in his own words: *Je ärmer eine Familie ist, einen desto grösseren Anteil von der Gesammtausgabe muss zum Beschaffung der Nahrung aufgewendet werden.*

In Copenhagen, however, the expenditure on food of the above mentioned families was found to be approximately 30 per cent in all of the three investigated income groups (3,000–4,000 Kroner, 5,000–6,000 Kroner, and 7,000–8,000 Kroner).

From a review of the literature on the subject it appears that Chapin [2] in his investigations in New York into the consumption of some 350 families discovered the same thing. He found that the proportionate expenditure on food was about 44–45 per cent in all the investigated income groups (from 500 to 1,200 dollars) and explains it in the following way: "The explanation lying in the exorbitant rent percentage (20–25 per cent) on low incomes, which makes adequate food expenditure (a high percentage) impossible." Chapin's families consisted of the parents and four to six children.

As regards Copenhagen, not only high rent but also high taxes, which increase with income, must be taken into consideration.

As Engel's work [3] is more than 70 years old I have looked it up in order to see how much it went into detail. It appears that his material, which comprised about 200 families, consisted partly of rural workers, and that the rent expenditure of all the workers' families was relatively low, while their taxes were purely nominal and their savings of no importance.

The exact figures of Chapin's observations in New York in 1907 are as in Table I.

For the Copenhagen families in 1922 the figures are those in Table II.

[1] *Budgetlaegning*, 1927, Kbhvn. J. Frimodt's Forlag.

[2] "Standard of Living of Working Men's Families in New York City 1907," quoted from Benjamin R. Andrews, *Economics of the Household*, 1925, p. 99.

[3] Reprinted in *Bulletin de l'Institut International de Statistique*, Tome IX, 1895.

TABLE I

THE AVERAGE PERCENTAGE EXPENDITURE OF 369 FAMILIES IN NEW YORK IN 1907

Income group in dollars	400	500	600	700	800	900	1,000	1,100	1,200
Expenditure on food	40.8	44.4	44.6	45.6	44.3	44.7	44.7	45.6	45.0
Expenditure on rent	26.8	25.9	23.6	21.9	20.7	19.0	18.1	16.2	19.8
Number of families investigated	8	17	72	79	73	63	31	18	8

In Engel's material, savings play such a small part that it does not matter whether the percentage consumption is calculated from the income or the consumption.

In the Copenhagen material savings are relatively so important that the percentages of nearly half of the families are substantially altered. For instance, in the case of Family X, a printer's family (current No. 2 in Stat. Med.), it is increased from 15 to 29, because with a yearly income of 5,263 Kroner this family saves as much as 2,600 Kroner, and with Family XVII (current No. 76 in Stat. Med.) the savings are 2,590 Kroner out of an annual income of 7,121 Kroner; so that in this case the percentage expenditure for food is increased from 20 to 31 per cent when the food consumption is calculated from the expenditure instead of from the income.

The Danish material is certainly very scanty, but, on the other hand, it is unusually uniform.

TABLE II

EXPENDITURE OF FAMILIES OF TWO MEMBERS (MAN AND WIFE) IN COPENHAGEN IN 1922 FOR FOOD, RENT AND DIRECT TAXES

Absolute Numbers

Current number	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV	XV	XVI	XVII	XVIII	XIX	XX
Annual income, Kroner	3,437	3,441	3,532	4,027	4,405	5,011	5,155	5,197	5,213	5,263	5,322	5,413	6,039	6,481	7,110	7,121	7,200	7,463	10,634	12,000
Annual expenditure, Kroner	3,749	3,297	3,554	3,442	4,476	5,118	3,749	5,095	5,440	2,663	4,535	4,234	6,511	6,071	6,795	6,648	4,610	8,424	8,551	9,703
Expenditure on food, Kroner	788	1,131	968	1,207	1,567	1,175	1,092	1,658	1,186	784	1,868	1,884	1,978	2,326	1,998	2,314	1,444	2,111	2,134	2,683
" " rent, Kroner	705	364	835	470	472	575	546	648	801	271	496	372	525	547	511	462	821	1,265	431	685
" " direct taxes, Kroner	299	276	269	50	157	485	306	481	354	504	421	707	522	350	756	815	652	720	1,005	1,127
	Percentage of Income																			
Expenditure on food	23	33	27	30	36	23	21	32	23	15	35	35	33	36	28	33	20	28	20	22
" " rent	21	11	24	12	11	12	11	12	15	5	9	7	9	8	7	6	11	17	4	6
" " direct taxes	9	8	8	1	4	10	6	9	7	10	8	13	9	5	11	11	9	10	9	9
	Percentage of Expenditure (Consumption)																			
Expenditure on food	21	34	27	35	35	23	29	33	21	29	41	44	30	38	29	35	31	25	25	28
" " rent	19	11	23	14	11	11	15	13	15	10	11	8	8	9	8	7	18	15	5	9
" " direct taxes	8	8	8	1	4	9	8	9	7	19	9	17	8	6	11	12	14	9	12	12

A NEW GRID FOR FITTING A NORMAL PROBABILITY CURVE TO A GIVEN FREQUENCY DISTRIBUTION [1]

By F. C. Martin, *Fukien Christian University*, and Dickson H. Leavens, *Harvard Graduate School of Business Administration*

The usual method of fitting a normal probability curve to a given frequency distribution is described in many text-books [2] on statistical methods. The arithmetic mean and the standard deviation of the given distribution are computed, and the maximum ordinate of the corresponding normal curve is found from the formula:

$$y_0 = \frac{N}{\sigma\sqrt{2\pi}}$$

where σ is measured in terms of class intervals. Then with the aid of a table of ordinates of the normal curve, in percentages of y_0 corresponding to different multiples of σ, the normal curve is plotted over the graph of the original frequency distribution. To do this requires either the drawing of new horizontal and vertical scales with σ and y_0 replacing the original unit of measure and the original frequency scale respectively; or else the computation of abscissas and ordinates of the normal curve suitable for plotting on the old scale by multiplying the tabular values by σ and y_0 respectively. Either of these methods is tedious.

Galton [3] suggested, and Hazen [4] and Whipple [5] developed, the use of probability paper,[6] so ruled that when the *cumulative* normal probability curve, or ogive, is plotted it will be a straight line. By plotting any given cumulative frequency distribution on probability paper, it is possible to observe how closely it approximates to a straight line and thus to get some idea of how nearly normal it is. Such paper is convenient for many purposes, as, for example, the graphic location of median and quartiles, and the estimation of frequencies between given

[1] Part of the method here described was developed in a paper by F. C. Martin, "The Use of a New 'Logarithmic-Squared' Chart in Applying the Equivalent Normal Frequency Curve to a Given Distribution," *Proceedings of the Natural History Society of Fukien Christian University*, Foochow, China, Vol. 2, pp. 28–37, June, 1929.

[2] See, for example, Yule, *An Introduction to the Theory of Statistics*, 2d ed., London, 1912, pp. 307–308; or Rugg, *Statistical Methods Applied to Education*, Boston, 1917, pp. 210–211.

[3] Francis Galton, "A Geometric Determination of the Median Value of a System of Normal Variants, from Two of Its Centiles," *Nature*, Vol. 61, pp. 102–104, November 30, 1899.

[4] Allen Hazen, "Storage to be Provided in Impounding Reservoirs for Municipal Water Supply," *Transactions of the American Society of Civil Engineers*, Vol. 77, pp. 1539 ff., 1914.

[5] G. C. Whipple, "The Element of Chance in Sanitation," *Journal of Franklin Institute*, Vol. 182, pp. 37–39 and 205–227, July and August, 1916.

[6] Probability paper is obtainable from the Codex Book Company, Norwood, Massachusetts.

limits. However, when it is desired to compare a given distribution with a normal one, the eye can see this relation more readily from non-cumulative curves than from cumulative ones. Moreover, probability paper exaggerates departures from normal at the extremes of the distribution. Hence, probability paper, useful as it is, cannot entirely replace the need of fitting normal curves to data.

This need suggests an attempt to make a grid on which the normal curve itself will appear as a straight line. Given the equation of the normal curve:

$$y = y_0 e^{-\frac{x^2}{2\sigma^2}} \tag{1}$$

we take the logarithms of both sides,[1] getting

$$\log y = \log y_0 - \tfrac{1}{2}\left(\frac{x}{\sigma}\right)^2. \tag{2}$$

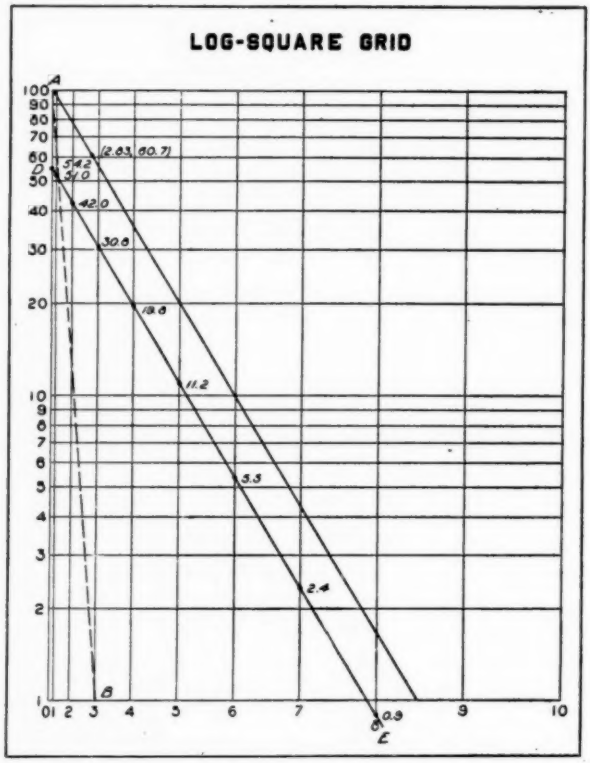
This equation is linear in $\log y$ and $\left(\frac{x}{\sigma}\right)^2$. Therefore, let us construct a grid, Chart I, in which the horizontal scale is marked off at distances from the origin to the right equal to 1^2, 2^2, 3^2, 4^2, . . ., but labelled 1, 2, 3, 4, . . ., and in which the vertical scale is logarithmic. If from a table of ordinates of the probability curve we plot on this grid the values of $\frac{x}{\sigma}$ and of $\frac{y}{y_0}$, the latter as percentages, we shall have a straight line, AB, passing through the point (0, 100), which will represent the vertex of the curve, while the remainder of the line corresponds to the right-hand half of the curve. The vertex and one other point are sufficient to determine this line. The most convenient point to use is (1, 60.7), since σ has been taken as the unit for x, and $\frac{y}{y_0}$ always equals 60.7 per cent at the point corresponding to $x = \sigma$. More generally, if x is measured in terms of some other unit than σ, the second point will be (σ, 60.7). If ordinates corresponding to actual values of y instead of to $\frac{y}{y_0}$ are used, the straight line will be parallel to the line through (0, 100) but will pass through (0, y_0) instead. Thus, for a given σ and y_0, it is easy to draw a straight line which represents the corresponding normal probability curve.

We might now plot on this grid the actual points of a given frequency

[1] Buckner Speed, in two papers, "A New Graphic Method of Using the Normal Probability Curve," *Proceedings of the Casualty Actuarial Society*, Vol. 2, pp. 120–122, 1915, and "Note on the Normal Probability Curve," *Ibid.*, Vol. 12, pp. 114–116, 1925, takes the logarithms of both sides and points out that (2) is a parabola when plotted on a semi-logarithmic grid, but he does not attempt to convert the curve into a straight line.

distribution and compare them with the straight line. This method would have the same disadvantage as cumulative curves on probability paper, because the eye could not readily interpret the relationship on account of the distortion. A more fruitful method is to obtain from the grid the coördinates of points of the normal curve and then plot them on the original graph of the given distribution. For this purpose a "log-square" grid may be constructed once for all, and used repeatedly

CHART I



by drawing pencil lines on it.[1] Since the vertical scale is logarithmic, the straight line may be shifted up or down one bank if it does not give the desired intersections in its original position.

The steps in the process may be illustrated by working out an example. Table I and the column diagram in Chart II represent the frequency distribution of the number of grains per plant in 384 plants of summer rice, as counted by Professor Claude R. Kellogg of Fukien

[1] The writers have had the grid of Chart I photolithographed in a size of about 10 x 12 inches, and will be glad to send a sample to anyone interested.

TABLE I

FREQUENCY DISTRIBUTION OF NUMBER OF GRAINS PER PLANT OF SUMMER RICE *

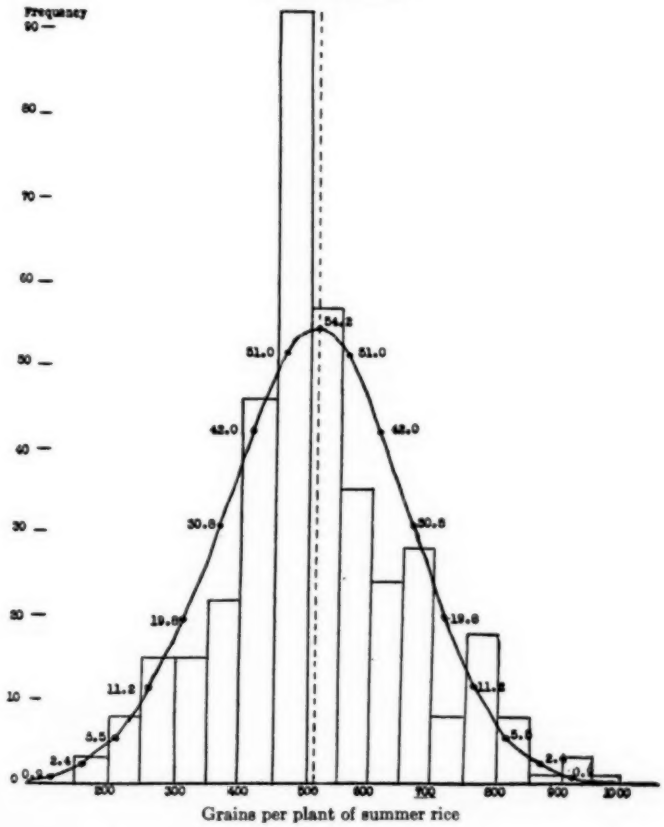
Number of grains	Frequency	Number of grains	Frequency
150–200	3	600–650	24
200–250	8	650–700	28
250–300	15	700–750	8
300–350	15	750–800	18
350–400	22	800–850	8
400–450	46	850–900	1
450–500	92	900–950	3
500–550	57	950–1000	1
550–600	35		N=384

* As counted by Professor Claude R. Kellogg.

Christian University. The arithmetic mean is 515 grains, and the standard deviation is 2.83 class intervals, or 142 grains.

CHART II

(Data from Table I)



1. Compute $y_0 = \frac{N}{\sigma\sqrt{2\pi}}$

$$= \frac{384}{2.83 \times 2.507}$$

$$= 54.2.$$

2. On the log-square grid, Chart I, locate the points (0, 100) and (2.83, 60.7), and connect them by a straight line, AC. This line represents the right-hand half of a normal curve with $\sigma = 2.83$ class intervals and $y_0 = 100$ per cent or unity.

3. Through the point (0, 54.2) draw a line, DE, parallel to the above line. In effect, since the vertical scale is logarithmic, this amounts to multiplying all the ordinates of AC by 54.2. The new line, therefore, represents the right-hand half of a normal curve with $\sigma = 2.83$ class intervals and $y_0 = 54.2$.

4. On Chart II, draw an ordinate through the mean, 515, as a new Y-axis from which to measure. On this new axis plot the point (0, 54.2). Then from Chart I, read off on the vertical logarithmic scale the value of the intersection of the straight line DE with the ordinate labelled 1. Plot this value, 51.0, on Chart II at a distance of 1 class interval to the right and also to the left of the new Y-axis. This can easily be done by eye or with the aid of a scale, since Chart II is already ruled in class-interval units, so that the shift of origin is the only new element. Similarly read off the value of the intersection with the ordinate labelled 2, and plot it, 42.0, on Chart II at a distance of 2 class intervals to the right and left of the new Y-axis; and so on, until sufficient points are plotted. These points connected by a curved line give the normal curve equivalent to the original column diagram: that is, having the same mean, the same standard deviation, and the same area.

5. To check the area: Table II gives the ordinates as read from Chart I. They[1] total 381.4. The tails of the curve would account for about

TABLE II

COÖRDINATES OF NORMAL CURVE FITTED TO DISTRIBUTION OF TABLE I

Abscissas: Grains per plant	Ordinates: Frequency	Abscissas: Grains per plant	Ordinates: Frequency
115	0.9	565	51.0
165	2.4	615	42.0
215	5.5	665	30.8
265	11.2	715	19.8
315	19.8	765	11.2
365	30.8	815	5.5
415	42.0	865	2.4
465	51.0	915	0.9
515 (mean)	54.2		381.4

[1] This amounts to finding the area by the trapezoidal rule; Simpson's rule gives approximately the same result, 381.3.

1 more, making 382.4 for the area of the curve, as compared with 384 for the original distribution. This shows a discrepancy of 1.6 or less than one-half of one per cent, which is as good as could be expected for a graphic method.

EMPLOYMENT STATISTICS AS MEASURES OF UNEMPLOYMENT

By A. C. C. Hill, Jr., *The Brookings Institution*

Statistics of employment [1] are often interpreted as indices of unemployment on the theory (a) that during a period when employment falls, unemployment rises, and (b) that during a period when employment rises, unemployment falls.

The following tables, based on British experience, provide standards against which these assumptions may be checked when applied to specific industries and not to industry as a whole. The figures contained in the tables [2] show the number of unemployed workers and the number of employed [3] workers in various industries as of the end of July of each year, except 1926, from 1923 to 1929. The year 1926 has been omitted because of the exceptional circumstances prevailing in that year.[4] The net change between 1923 and 1929 is also indicated.

The assumption that declining employment is accompanied by increasing unemployment is supported by the records of certain specific industries. The bleaching, woolen and worsted, and coal-mining industries, depicted in Table I, are cases in point.

TABLE I

NUMBER EMPLOYED, NUMBER UNEMPLOYED, AND NET CHANGE IN CERTAIN BRITISH INDUSTRIES, JULY 1923 TO 1925, AND 1927 TO 1929

(In thousands)

	1923	1924	1925	1927	1928	1929	Net change 1923–1929
	Textile Bleaching, etc.						
Employed	103	103	102	108	100	98	− 5
Unemployed	14	14	17	13	17	18	+ 4
	Woolen and Worsted						
Employed	248	242	192	225	206	201	− 47
Unemployed	25	19	56	24	37	38	+ 13
	Coal Mining						
Employed	1,229	1,174	1,056	941	791	872	−357
Unemployed	38	86	184	258	325	203	+165

[1] Such as those collected and published by the United States Bureau of Labor Statistics, the New York Department of Labor, the Massachusetts Department of Labor and Industries, and the Department of Research and Statistics of the Philadelphia Federal Reserve Bank.

[2] Data from the *Labour Gazette*, August and December of the years concerned.

[3] Derived by subtracting the unemployed from the insured.

[4] The seven months coal strike and the general strike.

Figures for other industries, included in Table II, support the assumption that rising employment is accompanied by declining unemployment. For example, in the building trades a rise of 124,000 in employment has been accompanied by a decrease of 15,000 unemployed. A similar condition is to be noted in the other industries included in the table.

TABLE II

NUMBER EMPLOYED, NUMBER UNEMPLOYED, AND NET CHANGE IN CERTAIN BRITISH INDUSTRIES, JULY 1923 TO 1925 AND 1927 TO 1929

(In thousands)

	1923	1924	1925	1927	1928	1929	Net change 1923–1929
	Building						
Employed	630	651	696	788	732	754	+124
Unemployed	87	71	59	60	85	72	− 15
	Printing and Paper						
Employed	331	341	347	363	359	373	+ 42
Unemployed	22	19	18	16	15	14	− 8
	Electrical Engineering						
Employed	57	68	73	75	76	81	+ 24
Unemployed	4.5	3.5	4.3	3.8	4.0	3.2	− 1
	Automobiles and Aircraft						
Employed	175	187	201	215	213	227	+ 52
Unemployed	19	16	14	18	22	18	− 1
	Road Transport						
Employed	121	128	137	156	156	164	+ 43
Unemployed	26	22	19	17	19	19	− 7
	Cotton						
Employed	453	471	512	516	470	475	+ 22
Unemployed	123	91	65	54	84	80	− 43
	Ship Building						
Employed	155	183	158	168	145	158	+ 3
Unemployed	118	72	84	48	57	47	− 71

The employment figures would have been of little use in determining the quantity of unemployment since the increases in employment in the various industries are not comparable to the various decreases in unemployment. For example, employment has increased in the cotton industry by 22,000 while unemployment has decreased by 43,000,

whereas employment in electrical engineering has increased by 24,000 while the numbers of unemployed attached to the industry have diminished by only 1,000.

Similar comment may be made in connection with the data in Table I.

In other important industries, however, the assumption that when employment falls, unemployment rises does not hold. Table III, for

TABLE III

NUMBER EMPLOYED, NUMBER UNEMPLOYED, AND NET CHANGES IN CERTAIN BRITISH INDUSTRIES, JULY 1923 TO 1925 AND 1927 TO 1929

(In thousands)

	1923	1924	1925	1927	1928	1929	Net change 1923–1929
	General Engineering						
Employed	538	536	549	545	526	534	− 4
Unemployed	137	91	78	55	56	53	−84
	Docks, Harbors, Rivers, etc.						
Employed	141	147	134	141	116	119	−22
Unemployed	53	49	58	40	55	52	− 1
	Government						
Employed	393	374	385	375	354	364	−29
Unemployed	35	31	30	25	27	29	− 6
	Brass and Allied Metal Wares Manufacturing						
Employed	26	25	26	27	25	26	0
Unemployed	7	5	4	3	3	2	− 5

example, gives evidence of the fact that just the reverse prevails in certain industries. Indeed in these industries and others, unemployment has decreased at the same time that employment has decreased.

In still other important industries the assumption that when employment rises unemployment falls, fails of substantiation. Table IV gives evidence of the fact that in these industries the opposite is true and that unemployment has increased at the same time that employment has increased.

TABLE IV

NUMBER EMPLOYED, NUMBER UNEMPLOYED, AND NET CHANGE IN CERTAIN BRITISH INDUSTRIES, JULY 1923 TO 1925 AND 1927 TO 1929

(In thousands)

	1923	1924	1925	1927	1928	1929	Net change 1923–1929
	Distributive Trades						
Employed	1,187	1,275	1,374	1,511	1,530	1,588	+401
Unemployed	75	77	84	70	84	91	+ 16
	Public Works Construction						
Employed	104	114	127	144	130	133	+ 29
Unemployed	24	21	22	25	30	31	+ 7
	Silk and Artificial Silk						
Employed	36	40	44	51	66	68	+ 32
Unemployed	2.6	1.6	2.9	3.6	4.0	5.7	+3.1
	Tramway and Omnibus Service						
Employed	107	116	117	132	137	151	+ 44
Unemployed	3.4	3.0	3.8	3.3	3.6	3.6	+ .2
	Brick, Tile, etc. Making						
Employed	57	67	74	83	72	75	+ 18
Unemployed	4.7	4.0	4.5	4.3	8.4	6.6	+1.9

PUBLIC UTILITY PROBLEMS

A dinner meeting of the American Statistical Association was held on Wednesday evening, January 28, 1931, at the Aldine Club, 200 Fifth Avenue, New York City. Three hundred and one persons were in attendance. Colonel Malcolm C. Rorty presided. The first speaker of the evening was Martin J. Insull, President of the Middle West Utilities Company. In speaking on the subject, "Building and Financing a Utility System," he outlined the growth of the Middle West Utilities Company, using that organization as an example of the evolution taking place in the field of electric light and power production. This company had its inception in 1910, when Samuel Insull organized the plants in this line of business, combining all the towns of Lake County, Illinois. He discovered that, by thus consolidating, both production costs and selling rates for electricity could be reduced, even though the amount of capital invested was increased. This combination proved to be a forerunner of the present great holding companies now controlling a large proportion of the electric power production in the United States. This process of consolidation has made possible that cheapness of electrical energy which European investigators have recently cited as being one of the principle bases of American industrial supremacy.

The growth of the Middle West Utilities Company has been phenomenal. At the end of 1912, its first year of operation, it served less than 300 communities in two states and did a gross business of about $3,680,000. By the close of 1930, it had so expanded that it served some 6,600,000 people in 4,624 communities in thirty states, and its gross business amounted to more than $180,000,000. Of the 4,624 communities served, 1,408 had never enjoyed electrical service until the Middle West Utilities Company placed it at their disposal.

In 1930, this company had 32,235 employees. To each of these at Christmas time, the company gave a paid-up life insurance policy ranging in value from $500 to $1,500 according to the length of time the employee had been with the company. The total amount of insurance thus furnished to employees was $27,000,000. In addition, the employees carried, at their own expense, about $39,000,000 more. Recently, an investment company has been organized for the benefit of employees.

Incidentally, the company operates some 321 ice plants, having a capacity of 14,726 tons daily. Probably, no other company owns as many ice plants.

In the process of consolidation, the company has found it feasible to close 476 obsolete electrical generating stations. Between 1913 and 1930, it increased its volume of electrical output from 100,000,000 kilowatt hours to 4,396,000,000 kilowatt hours. It also exchanges current with many other companies. Since the peak load for some companies is at one hour of the day and for other companies at another hour, this interchange enables all the companies concerned to produce at a lower rate because of decreased investment. In 1913, it took 5.1 pounds of coal to generate one kilowatt of electricity. By 1930, efficiency had been so greatly increased that only 1.82 pounds of coal were required. This increase in efficiency permitted a marked reduction in the prices charged for

electricity during the same period in which prices of most other commodities were rising sharply.

The general policy of the company has been to have the various operating companies financed on a basis of approximately 50 per cent bonds, 25 per cent preferred stock and 25 per cent common stock. In the holding company, the policy, in normal times, is to issue all stock and no bonds. At the present time, however, the Middle West Utilities Company has outstanding, not only $142,000,000 of common stock and $60,000,000 of preferred stock, but also $50,000,000 in notes. These were issued recently when money was needed for purposes of expansion and it was thought impracticable to try to sell stock. A number of subsidiary holding companies have been organized, the reason for their existence being the necessity of supervising offices for groups of operating companies in states distant from the offices of the Middle West Utilities Company. It has also been found that they are a distinct aid in raising capital for this large utility system. At present, the capital of the Middle West Utilities Company in the hands of the public has a par value of about $1,140,000,000, while the market value is around $1,325,000,000. By no stretch of the imagination can it be said, legitimately, that there is more than 5 per cent of "water" in all these security issues combined, though Mr. Insull did not concede there was even this amount. The policy of the company has always been to allow the operating companies a high degree of autonomy. The company holds that it is the right and duty of the State Commissions to regulate the business of the operating companies since these companies are local monopolies. On the other hand, it has consistently contended that there is no excuse for regulating the business of holding companies as they are non-monopolistic and not "affected with a public interest." The best interests of the public are served by regulated operating companies, but with holding investment companies as free as any other private business that gives its attention and support to properties in which its funds may be invested.

The second speaker of the evening was Dr. John Bauer, Director of the American Public Utilities Bureau. He discussed "Problems of Regulation as Affecting Consumers and Investors." Dr. Bauer began by stating that there is no question but that public utility organizations, as at present constituted, have energetically and efficiently developed the use of electrical power in industry, and it is sold to companies engaged in manufacturing or transportation, at surprisingly low rates. The truth seems to be, indeed, that these rates are too low, for their cheapness results in placing most of the expense of production upon the shoulders of the domestic users. In recent years, rates to large consumers have fallen faster than the cost of production. On the other hand, rates to the smaller consumers have not fallen as fast as the cost of production. Lower rates would, of course, stimulate consumption by domestic users. That this is true, is evidenced by the fact that the average New York household uses only about one-third as much electricity as the average household in Ontario, where rates are extremely low.

Throughout the United States, public utilities are nominally subject to regulation, but, in most cases, the commissions are averse to standardizing rates.

They act only in special cases. The difficulty is that no scientific system of regulation, no satisfactory yardstick, has as yet come into use. Different sets of experts advocate different systems of rate-making. What we need most is a definite system, clearly understood by everyone, which will continue year after year without change.

Such a system is, perhaps, more important from the standpoint of investors in public utility stocks, than from the standpoint of consumers of electricity. Government bonds fluctuate but little in value because the income from them is reasonably certain. The securities of public utility companies ought likewise to have their income practically guaranteed, so that they would be nearly as stable in value as government bonds. This would be entirely feasible because of the fact that the sales of electricity vary but little from year to year, and hence the public utility companies ought to have relatively stable incomes. Despite this stability in the industry, however, public utility stocks, at the end of 1929, were worth less than half their value at the high point of the year. The same degree of variation was found in the year 1930. One of the chief difficulties is that leaders in public utility finance have been more interested in stock speculation than in the business of producing electricity.

The third speaker of the evening was Thomas F. Woodlock, Editor of the *Wall Street Journal* and former Interstate Commerce Commissioner. He spoke on "Public Ownership." He agreed with Dr. Bauer that rates for electricity to large users of power are very low, but he did not feel that rates to domestic consumers are high on the average, being only $0.06 per kilowatt hour or $31.00 per annum for a household. Mr. Woodlock doubted greatly that the recent agitation against the power trusts has its origin in any discontent caused by the rates to domestic consumers. Obviously, it would be impossible, by any reduction of rates, to save any considerable amount of money for the average consumer when the total for the year is only $31.00. It appears, therefore, that the agitation has been artificially fomented by politicians interested, not in rates, but in votes.

He did not feel that the lack of definite regulation by the commission is due, primarily, either to indefiniteness in standards or to inability to know how to proceed. The real trouble is that the will to act is not present. There is too much politics and too little economic science applied.

Is the quest directed toward cheaper rates and better service? If so, there is nothing whatever to indicate that these ends can be obtained by substituting public for private ownership. Public ownership in Ontario is always being cited as an example. The fact is that Ontario is in an exceptional position and by no means can be considered typical. Furthermore, rates in Ontario are cheap only near the power plants. If we really desire a logical solution of the electric power problem, the best procedure to advocate is, contractual agreement on a profit sharing basis. The rate scale ought to be worked out on such a basis as to allow a company to increase its profits when it lowers the price of its product. This system has been worked out in Washington, D. C., and in a few other places and has resulted in a marked diminution in the prices of electricity, at the same time giving larger profits to the company furnishing the electricity.

Apparently, however, most antagonists of the power companies are not, primarily, interested either in cheap rates or good service. They object mainly to the fact that companies are making good profits. Some are anxious for government control mainly because they believe it would educate officials in the art of management.

Mr. Woodlock stated that his own objection to public ownership was largely that he disliked bureaucracy, and the more government ownership, the more bureaucracy we shall have. The only justification for regulating public utilities lies in the fact that they are monopolies.

The last regular speaker of the evening was Mr. Jacob J. Goetz of Whitman, Ransom, Coulson, and Goetz. His topic was "Private Ownership." He mentioned that his own practical experience in connection with rates was that the Commissions did not fix rates high enough to afford a return upon present *fair value* of property, and that the utility companies did not embark upon rate litigation when the rates were sufficient to afford a reasonable return upon *investment*.

If a standard other than "fair value" is to be set up for the valuation of public utility property, because the term "fair" is claimed to be elastic, he raised the question whether "prudent investment" with so indefinite a qualification as "prudent" could serve as an acceptable substitute.

Mr. Goetz stated that the idea was prevalent that regulation was predicted upon the monopolistic character of an industry or the grant of a special franchise to use the public highways. He asserted, however, that both the basis and scope of governmental regulation in this country are much broader, being based upon the accepted doctrine that when private property is affected with a public interest, it ceases to be *juris privati* only, and that the far-reaching consequences of principles and policies evolved for public utilities are hardly appreciated. This doctrine of public interest has been applied to warehouses located on private property and not excercising a public franchise, to the insurance business distinct from public use of property, to rental of dwellings under conditions of emergency, and to market agencies irrespective of the amount of capital employed. There is now a bill in the Legislature proposed, on the same grounds, to regulate press news agencies.

Furthermore, there is a political disposition toward the creation of governmental agencies to carry on public utility services, with power to issue securities, whose operations might not have the advantage of the federal constitutional guarantees available to privately owned public utilities.

The discussion was led by Thatcher C. Jones of New York University. He raised a number of questions which he believed were worthy of discussion. Among them were: Does the interchange of electrical energy by different corporations indicate that we need federal rather than state control of public utilities? Is it not true that the Middle West Utilities Company is so built up as to give control to a very small amount of stock?

Professor Jones called attention to the fact that low electric rates to large users may be justified on a promotional basis and that, furthermore, these low rates are justifiable because a substantial part of the power is used at a time when the load on the generator is not at its peak.

He expressed the view that, even if we had definite and stable rates guaranteed by regulatory bodies, one could not expect the same stability in the price of public utility stocks that is found in the price of government bonds.

In regard to Professor Jones' query concerning the Middle West Utilities Company, Mr. Insull stated that it was by no means true that the control was in the hands of a small group of stockholders. At present, the Company has 82,000 stockholders, no small group being dominant.

The meeting adjourned.

WILLFORD I. KING, *Secretary*

PROSPECTIVE CHANGES IN THE POPULATION OF THE UNITED STATES

A dinner meeting of the American Statistical Association was held on Tuesday evening, February 24, 1931, at the Aldine Club, 200 Fifth Avenue, New York City. Eighty-seven persons were in attendance. Dr. Henry Pratt Fairchild of New York University acted as Chairman.

The first speaker of the evening was Guy Irving Burch from the Population Reference Bureau of New York University. The subject of his address was "Changes in Factors Affecting Population Growth." Mr. Burch outlined a number of the important trends which are influencing the size and composition of the population. He pointed out that the present trend of the birth rate is sharply downward, not only in the United States but in all of the more progressive nations of the world. In Germany, England, Sweden, France, and the United States, the birth rate has now declined to less than twenty per thousand. In the same nations, the general tendency is for the death rate to hover around twelve per thousand. At present, therefore, despite the decline in both the birth and death rates, there is a very considerable excess of births over deaths. The lowness of the existing death rate is, however, caused by the fact that, in recent decades, population has been rapidly increasing. The chances are that the death rate in the future will actually advance. On the other hand, one cannot say how much lower the birth rate may go. It is, therefore, not at all impossible that, in the relatively near future, we may find deaths exceeding births both in western European nations and in the United States.

Until the opening of the World War, the population of the United States was increased by four great waves of immigration, each rising higher than the one preceding. The waves before 1895 were made up largely of immigrants from Northwestern Europe. The great wave having its culmination in the period 1908–1914 was, however, dominated by immigrants from Southern and Eastern Europe. The World War caused a great decline in immigration, but, as soon as it was over, the influx from abroad again began. In 1921, Congress passed the first law restricting the quantity of immigration into the United States, limiting the number of "quota" aliens of any European nationality to 3 per cent of the number of foreign-born persons of such nationality resident in the United States as determined by the Census of 1910. In 1924, Congress passed a new act

which contained both a temporary and permanent basis of numerical limitation of quota immigrants coming from European countries. The more permanent basis limited the annual "quota" of any European nationality to a number which bears the same ratio to 150,000 as the number of white inhabitants (native born as well as foreign born) having that national origin bears to the number of inhabitants in Continental United States in 1920. The temporary basis of the 1924 Act was of the same principle as that of the 1921 Act, only the per cent was changed from 3 to 2, and the census data of 1890 were used instead of those of 1910. The temporary basis went into effect July 1, 1924, and was supplanted by the national origins basis in 1929. The present law gives Northwestern Europe about five "quota" immigrants to one for Southeastern Europe. In the fiscal year ending June 30, 1930, the tabulated net population increase caused by the excess of immigration over emigration was 173,789, or less than one-fourth as much as the corresponding number in 1913.

Mr. Burch pointed out that diminished immigration and the falling birth rate would probably result in the population of the future having within the reproductive age group a percentage of its members smaller than that now existing. One might expect that this reduced percentage would make the birth rate decline still further. This is not, however, a necessary implication, for the average number of births per woman might be increased since, at present, the birth rate is governed, not by reproductive capacity, but by the standard of living. While in the future children will probably constitute a percentage of the population smaller than that of today, more of them are likely to reach maturity, and they will have improved health and education.

The most serious threats to progress in progressive nations, at the present time, are war and the dysgenic differential birth rate. War tends to destroy the stronger sections of the population. The fact that the birth rate of the least competent sections is highest of all tends constantly toward the deterioration of our population. It is to be hoped that increasing knowledge of birth control among the lower strata of the population will tend to make the birth rate of the future less dysgenic.

The second speaker on the program was Dr. Louis I. Dublin, Statistician of the Metropolitan Life Insurance Company, who spoke on "Changes in the Age Composition of the Population." He began by enumerating dynamic factors bringing about changes in the structure of the population as follows: (1) migration, (2) births, (3) deaths, (4) marriages, and (5) divorces. He pointed out that not only is the structure of the population affected by these forces, but the structure, in turn, affects the rate of growth of the population. For example, the birth rate is, to a considerable extent, dependent upon the proportion of the population between the reproductive age limits.

In the United States, the proportion of the population in the younger ages has diminished greatly since 1850. It is, however, still much greater than that characterizing the population of France or of Austria.

The most important reason for the changes which have occurred in the composition of our population is the fall in the birth rate. At the beginning of the last century, the birth rate was probably not far from forty-five per thousand, or

more than double what it is at present. During the same period, the death rate has fallen from around twenty-five to the present level which is about twelve.

Another recent force causing a change in the composition of our population has been enforced reduction in immigration. For the most part, immigrants have been young people. When, therefore, immigration falls, the proportion of people below middle age tends to decline.

The probabilities are that, in the near future, the rate of natural increase will diminish. The death rate is likely to rise, not because health conditions are growing worse, but because we are having a larger proportion of old people in our population. Thus far, we have made no progress in increasing the ultimate span of human life. As long as this is true, every diminution in the death rate in earlier years is offset by an increase in the death rate of persons in the higher ranges of life.

The birth rate will probably continue to fall for some time but it is not likely that the decline will go on indefinitely.

The increase in population between 1920 and 1930 was at the average rate of 1.6 per cent per annum, a rate of growth which would double the population every forty years. Present tendencies will, however, reduce this rate of growth without, as a matter of fact, changing materially the percentage of the population between twenty and fifty years of age. There will be a marked diminution in the proportion of the population under twenty and a marked increase in the proportion of the population over fifty. This change in age composition will presumably make our nation more conservative. There will be a relative increase in the demand for goods used by adults and a decrease in the demand of goods for children. There will be fewer young dependents and more old ones.

Since the death rate of females is materially less than that of males at ages over about thirty-five, we can look forward to having a considerable excess of females in our population, especially in the older age groups.

The third speaker of the evening was Dr. Alfred J. Lotka who reported on his mathematical studies of "The Trend of the Birth Rate in a Population Growing According to the Logistic Law." He began by pointing out that the Pearl-Reed logistic curve has fitted the growth of the United States exceedingly well right up to the date of the last census. For this reason, and because of the central position which the logistic curve has come to occupy in population statistics, special interest attaches to a study of the typical characteristics of a population growing in accordance with this law. Such a population has a definite pattern. Its birth rate, death rate, age distribution and other demographic characteristics are not capricious, but conform more or less closely to a model or type determined by the logistic law of growth itself. By a suitable mathematical procedure, it is possible to compute these characteristics for a model population and to compare the results with the actual population of the United States. The results of such computation were presented in a series of graphs which showed more particularly how the birth rate in a logistic population declines first slowly, then rapidly, and finally again slowly toward a constant level; how the age distribution in the meantime passes from an initial form corresponding essentially to the compound interest law of increase, to an ultimate population corresponding to

a life table age distribution. Between these two extremes, the population passes through transitional stages such as that which prevails at the present time. The result of the computations for a model population were compared, after making certain suitable allowances, with the corresponding features in the actual population of the United States. A close agreement was found between the calculated birth rates and the observed birth rates in past years and also between the calculated age distribution and the age distribution of the native whites at the time of the last census (1920).

To make forecasts on the basis of the material presented is difficult, and the speaker referred to these forecasts only incidentally without laying any emphasis on them.

The last speaker of the evening was Dr. O. E. Baker of the United States Bureau of Agricultural Economics. He spoke on "The Food Supply of the Future." Dr. Baker presented a chart showing the rapid rate of growth of the population and the comparable aggregate growth of agricultural production. The chart indicated that the general tendency has been for agricultural production to lag behind population since the base period 1897–1901, which was a period of extraordinary opulence. However, agricultural production has spurted upward occasionally, the peaks attained at such times touching the population trend. Such was the case in 1914 and also in 1926. In the years 1909, 1917, and 1921, agricultural production was far below the population trend. Since 1926, the trend of agricultural production has been somewhat downward, as contrasted with the continued upward movement of population. This decline is owing largely to adverse weather conditions.

Increasing production per acre: From 1921 to 1926, however, we witnessed the strange spectacle of increasing agricultural production per acre without any corresponding increase in the average crop yields per acre. This paradox is explained by the fact that more efficient methods of production have been utilized, especially from the standpoint of land area. The introduction of the gasoline engine has made it possible to use for meat and milk animals the feed formerly consumed by draft animals. As a result, we now produce 30 per cent more animal products than at the close of the War, with only a very slight increase in production of feed other than that released by the decline in horses and mules. The indications are that this 30 per cent increase has been obtained with an increase in feed of less than 20 per cent, the gain being due in part to the improved quality of the stock, better feeding practices, fewer deaths and, in some classes of stock, slaughter at an earlier age.

We now have no more hogs, for instance, according to the estimates, than fifteen years ago, but produce a third more pork and lard. This remarkable change is owing largely to increase in the number of pigs per litter raised (i.e., reduced infant mortality) and to reduction of disease later in life, also to better feeding methods, especially use of minerals and legumes, which have increased the production of meat per bushel of corn. Another reason for increased efficiency in production is that we are now growing a larger proportion of live stock than formerly in the North where the average quality of the animals is better and methods are more efficient than in the South.

Less important, accounting for only about 20 per cent of the increase in agricultural production, are the shifts from the less productive toward the more productive crops per acre, and from the less productive toward the more productive animals per unit of feed consumed. A shift occurred from wheat toward corn in the central Corn Belt, from corn toward cotton in the South, and from hay and grain toward fruit and vegetables in California and elsewhere. Likewise the number of beef cattle, which require eleven acres of crops to produce as many calories of food as that produced from three acres fed to hogs or dairy cows, has remained about stationary (after allowing for the cycle), whereas the number of dairy cows and hogs has increased.

Consumption per person: Changes in diet have accompanied the changes in production. A great decrease in consumption of cereals per person has occurred —from about 340 pounds in the pre-war years 1909–1914 to 240 pounds in recent years. This has been partially balanced by increased consumption of sugar, which has come mostly from our insular possessions. We are probably eating more vegetables, and, possibly, a little more fruit per person. We are eating much more milk and pork and, for several years past, less beef: in fact, the per capita consumption of beef in 1929 was lower than in any year since 1897 (when the index figures begin)—possibly smaller than in any year in our national history. The net result of the changes in diet, however, has been insignificant with reference to the per capita acreage of land required to feed the American people.

The population prospect: For some years now enrollment in the lower grades of the public schools has been stationary or declining. This is the result of a falling birth rate. It is a surprising fact that the course pursued by the birth rate in its decline has been practically the same in the urban as in the rural states. However, the decline in rural states started from a higher point and the birth rate remains considerably higher in these states. It is certain that unless the birth rate rises or the immigration laws are made more liberal, the population of the United States will reach a stationary condition of between 150,000,000 and 170,000,000 about 1960, and may thereafter decline. On the other hand, the factors that have tended to increase agricultural production seem likely to persist for several decades, and, in addition, it is becoming evident that it would pay, even at present prices of farm products, to use more fertilizer, especially on the better soils, while if prices should rise, there is as large an acreage available for crops as that in crops at present. It appears, therefore, that we shall have no difficulty in meeting the food requirements of our maximum population.

Outlook for food supply: Although, with the recovery from the economic depression, prices of farm products should rise somewhat, except possibly that of wheat, present indications do not justify the expectation of such a rise in prices as would have been justified had the former birth rate continued. The chances are that the exports of some agricultural products will soon increase rather than diminish.

Moreover, if the recent decline in the general price level should continue it would increase the debt which the farmers must carry. These conditions, in association with the prospect for continued motorization of agriculture and advances in technique, promise another decade of rapid changes in agriculture.

The changes seem likely to be hard on the old men who own farms, but helpful to the young men who may wish to buy them.

It also seems likely that with the improvement in weather conditions, the trend of production will soon be upward again, and that instead of population pressing on the food supply, as Malthus predicted 130 years ago, and was predicted more recently by the English chemist, Crookes, the food supply will continue to press on population.

The meeting adjourned.

WILLFORD I. KING, *Secretary*

THE STATISTICAL ASPECTS OF DISEASE

A dinner meeting of the American Statistical Association was held at the Fraternity Clubs, 22 East 38 Street, New York City, on Thursday evening, March 26, 1931, at 6 P. M. One hundred and fifteen persons were in attendance. The general topic of discussion was "The Statistical Aspects of Disease." The meeting was presided over by Dr. E. H. Lewinski-Corwin of the New York Academy of Medicine.

The first speaker of the evening was Dr. George Baehr, Attending Physician of Mount Sinai Hospital. He discussed the great importance of having a uniform classification and terminology covering diseases of all kinds. Various methods of classifying diseases have been in use for many years. Some of these classifications are based upon the cause of the disease, others upon the nature of the symptoms, and still others upon the anatomical location of the disease. Because of the use of these diverse methods, records of the diagnoses of physicians are often intelligible to none but the authors. For example, myocarditis may mean any true inflammation of the muscle of the heart no matter from what cause it may originate, or it may be employed for weakness of the heart muscle due to some remote cause. In understanding the nature of a disease, a knowledge of the cause is of prime importance.

At present, statistical records for infectious diseases are more accurate than are those for degenerative diseases which are the chief causes of sickness and death after middle age. We need equally accurate records for both, if effective efforts are to be continued for the prevention of disease and the prolongation of life.

A National Conference on Nomenclature of Disease, composed of representatives of all leading national societies of medicine, surgery and the specialties, of all governmental bureaus, and of the American Statistical Association, is now attempting to perfect a new classification of diseases, a classification which, from a scientific standpoint, is far superior to anything heretofore existing and which is sufficiently elastic to meet the needs of all types of hospitals and public health organizations, both large and small.

The second speaker of the evening was Dr. George Gray Ward, Professor of Gynecology and Obstetrics at Cornell University Medical College and Chief Surgeon of the Woman's Hospital, who spoke on the subject "Surgical Accounting." He began by stating that in the United States hospitals do not, as a rule, keep records sufficiently adequate to show the efficiency of the work being done in

the hospitals. Those in charge do not know what percentage of the patients are cured or what percentage die. They are utterly in the dark as to which physicians or surgeons are improving and which are growing worse in their results. They are unable to judge as to which treatments are efficient and which are inefficient, and whether the work of the hospital as a whole and of the individual surgeons measures up to the average standard which should be attained.

In Sweden, by contrast, each hospital is compelled to report to the government, every year, full data showing percentages of patients dying from each disease and information concerning the outcome of every case admitted. In cases of cancer, all patients are required to come to Stockholm, the Government paying the expenses of transportation. Every cancer case is kept under observation until the patient either recovers or dies.

In New York City, the Woman's Hospital is a pioneer in the field of surgical accounting. For eleven years, now, uniform records have been kept for all patients. To do this, it has been necessary to standardize procedures and define terms carefully. Death rates are calculated on the basis of the number of cases discharged from the hospital and on the number admitted. This procedure is fair, for death is a cause for discharge and is, therefore, comparable with other causes of discharge. Every patient readmitted to the hospital is considered a new case. Each surgeon is credited with the patients upon whom he operates. When an operation is performed under the direction of the Senior Surgeon who takes responsibility for the case, the case is credited to the Senior Surgeon. In those cases in which the Junior Surgeon is normally responsible, if he operates against his judgment on orders from the Senior Surgeon, he may place the responsibility for the case upon the Senior Surgeon by filing a formal protest against the operation. In order to prevent the record for a particular surgeon from being made to appear bad because of the fact that he received an undue proportion of hopeless cases, provision has been made that, when a surgeon deems a case to be practically hopeless, he shall call other physicians into consultation. When his judgment is confirmed, this case is placed in a special category. Blood transfusions and examinations under an anaesthetic, etc., are not counted as operations but merely as forms of treatment. By following the procedure just mentioned, it has been possible to chart the records of the respective surgeons and see exactly who is efficient and who is inefficient, who is improving and who is going down. The results of this study for the last eleven years clearly demonstrate that, in a hospital, a surgical audit is perhaps even more important than a financial one. And just as it is universally considered necessary to have chartered accountants audit the finances of an institution, it is certainly as essential that a real audit be made by competent statisticians of the surgical and medical results of the mortality and morbidity of an institution.

The third speaker of the evening was Dr. H. B. Logie, Executive Secretary of the National Conference on Nomenclature of Disease, who spoke on "The Scientific Basis of Morbidity Statistics." He pointed out that comparisons of morbidity rates for different times and especially for different places are seldom accurate. This incomparability is due to several causes. One is that it is extremely difficult to identify some causes. For example, we usually do not

differentiate influenza from the common cold until the epidemic becomes widespread.

Another reason for difficulty in comparisons is that the same disease is called by different names at different times and in different places. Even at the same time and place, different persons give different names to the same ailment. It is because of these facts that importance is attached to the new plan of standardizing the nomenclature and classification of diseases.

Diseases may be classified either according to the cause or according to the part of the body affected. The plan of classification now being worked out by the National Conference classifies each disease on both of these bases. The Dewey system is used separately for each basis. The numerals for the two classifications are entered on the same line separated by a dash. The first half of the code number gives the location of the disease in the body and the second half the causal classification.

One of the difficulties of classifying diseases according to cause is that different causes may give rise to the same group of symptoms. It is often hard to differentiate correctly between these causes, but a different cause signifies a different disease.

Unfortunately, physicians still distrust statistics. Their interest is centered on curing the patient rather than on keeping a record of what is happening. Too frequently they have found that theories apparently proved in one year by statistics, have been entirely upset by a new set of facts presented the next year. Only when the statistical records of disease are put upon a more scientific basis can this difficulty be overcome. This basis was defined as consisting of a volume of records and an accurate definition of specific terms.

The last regular speaker of the evening was Dr. George Draper, Associate Professor of Clinical Medicine in the College of Physicians and Surgeons of Columbia University. The topic which he dealt with was "The Relation of Constitution to Disease." He began by stating that physicians often think of patients merely as cases of disease, overlooking the fact that they are individuals. The latter point is extremely important, for disease is not an entity in itself, but is the expression of a conflict between the patient and some special buffet from environment. If all persons responded to the same environment in the same way, the characteristics of the patient might be ignored. As a matter of fact, however, the vulnerability of different persons to the same onslaught differs tremendously. Investigation has revealed the fact that it is possible to tell by observing a person's general form and personality make-up what type of disease he is likely to develop, and, indeed often, in what organ it will probably be located. Certain measurements alone, of the shape of the head, the jaw, face and body proportions will go far, in most cases, in establishing a diagnosis of such diseases as, for example, meningitis or pernicious anaemia.

The principle of correlations, upon which the study of total personality depends, is not a new one and has been used in various ways in the past and for various purposes, such as selecting proper people for given occupations. That capacity or quality of an individual which enables him to develop one or another disease is similarly a quality of personality. Consequently, for the purpose of

determining the disease potentiality of a person, it is necessary to consider, not only the anatomical structure of the body, but also the emotional patterns.

Another interesting aspect of the human being is the mosaic of masculine and feminine components in the personality. Every human being is partly masculine and partly feminine in both physical and psychological characteristics. Susceptibility to disease is dependent, to no small extent, upon the nature of this mixture. For example, gout is a masculine disease and only those women are likely to have gout who are strongly masculine in temperament. On the other hand, goiter or gall bladder disease is rarely found in men, except among those who have a well marked feminine component in their total personality.

Some diseases, such as rheumatic fever for instance, are found in individuals who display marked unevenness of development, both in the physical, emotional and mental spheres. An adolescent patient afflicted with this disease is likely to be an infant in some respects and an adult in others.

The discussion of the papers was begun by Dr. Charles F. Bolduan, Director of the Bureau of Health Education of the New York City Department of Health. He emphasized the extreme difficulty of persuading hospitals to keep adequate records of the patients coming under their jurisdiction. He stated that, as early as 1912, he discovered that hosiptal reports concerning patients were practically worthless and that, he, therefore, developed a plan whereby, without asking hospital physicians to adopt any particular classification of disease or any special nomenclature, uniform, comparable morbidity statistics could be compiled from all coöperating institutions. The essence of the plan was the introduction of a "discharge certificate," a certificate being sent for each patient to a central tabulating bureau and there coded and otherwise dealt with as death certificates are now in the Health Department. For eighteen years, he has striven to get the various hospitals to adopt this plan but lack of funds has thus far prevented its inauguration. This standardization of morbidity accounting for patients is a matter of greatest importance.

The last speaker participating in the discussion was Mr. George H. Van Buren of the Statistical Department of the Metropolitan Life Insurance Company. In discussing Dr. Logie's paper, he stressed the fact that the new nomenclature must be so shaped that it will present no insurmountable difficulties in the way of coding or tabulation technique. The speaker also said that it was vitally important that statistics based on the nomenclature shall be referrable at least, to those classified according to the title headings of the International List of Causes of Death. This is necessary in order that the records of morbidity may be checked, as far as possible, against those for mortality. The speaker said that the new nomenclature was such a daring departure from the form which nomenclatures had taken in the past that he at first feared that it could not be made practicable and workable. He was now confident, nevertheless, that it is entirely feasible to classify diseases both anatomically and etiologically and still not cause confusion in the statistics based upon such a system.

Two minor difficulties remain to be ironed out. First, the sickness of any patient may be due to two or more causes running concurrently. Shall it be classified as due entirely to one cause, or apportioned among several causes? In

hospital records, should the hospital base its classification of the case upon all the diseases from which the patient is suffering, or upon one particular disease treated at the hospital? Second, a problem arises from the fact that it is going to be extremely difficult to satisfy the demands of those who are working in the public safety field and who desire a wealth of statistical detail for use in their work. The speaker was confident, nevertheless, that Dr. Logie and those who are working with him would ultimately solve this problem.

The meeting adjourned.

WILLFORD I. KING, *Secretary*

PROGRESS OF WORK IN THE CENSUS BUREAU

THE FIFTEENTH CENSUS REPORTS

Volume I of the Fifteenth Census Reports was received from the printer May 12. It is an octavo of 1,268 pages. It contains a section or chapter for each state, giving the population of the state by counties and of each county by townships, precincts, election districts, or other minor civil divisions, as well as the population of all cities and other incorporated places in the state. The population of all cities of 5,000 inhabitants or more is given by wards. For the states and counties the land area and the population per square mile are also given. Chapters are included giving statistics in similar form for the various outlying territories and possessions (except the Philippine Islands, which were not covered by the Fifteenth Census).

At the beginning of the volume is a summary chapter which gives the total population of the United States by states, the urban and rural population by states, together with lists of cities, arranged, by states, in groups according to size.

Serious consideration was given to publishing all the volumes in octavo; but that format is not well adapted for the elaborate tables which are necessary in any complete or adequate presentation of census statistics. Volume I, however, which is limited to giving the total population of the various political units and does not include any elaborate tables can very well be printed in octavo, and as this is the volume for which there will be the greatest general demand, it was thought best to disregard uniformity for the sake of using that convenient size.

The complete final reports of the Fifteenth Census will probably comprise about 25 volumes as compared with the 13 volumes that made up the reports of the Fourteenth Census; and the number of printed pages will probably be more than twice what it was in the Fourteenth Census. This great increase results in part from the fact that this census covers some subjects not covered in the previous census, notably, unemployment and distribution; but to a greater degree probably, it is due to the fact that the statistics will be presented in greater detail. Certain tabulations will be shown for townships that were never shown before. Some new classifications have been introduced; for instance the classification of farms by type as mentioned below. There will be a tabulation of family data. Occupation statistics will probably be presented in greater elabora-

tion. In these and in other respects the scope and detail of the census have expanded to an extent that is almost appalling to those who are charged with the responsibility of completing the census before the end of 1932.

Publication of the Results of the Census of Agriculture

The first publication of the results of the Census of Agriculture was a series of state press releases—begun in August, 1930—giving the number of farms by counties as determined by a hand count of the farm schedules. This was followed by a summary for the United States, given out on October 15, which showed a total of 6,297,877 farms.

The results of the Census of Agriculture are being published in four series of state bulletins. One series presents farm data for townships, towns, precincts, or for whatever unit is the minor civil division of the state. This gives for each township or other minor civil division the number of farms, acreage of farm land classified according to its use, value of farm land and buildings, value of buildings, value of farmers' dwellings, and value of implements and machinery. There are approximately 50,000 minor civil divisions in the United States for which the above data will be shown. This is the first time that any farm statistics for minor civil divisions have been shown in the census reports since 1870. State bulletins in this series had been published for 28 states by June 1, and the Bureau expects to complete the series in the summer of the present year.

There will be three series of state bulletins presenting statistics by counties: The first of these series gives the number of farms, the total farm acreage and the acreage of crop land harvested classified by tenure; the acreage of various classes of farm land—crop land, pasture land, woodland, etc.; the number of farms, the farm acreage, and the acreage of crop land harvested classified by size of farm—under 3 acres, 3 to 9 acres, etc.; the value of farm land and buildings with separate items for land, for buildings, and for the farmers' dwellings, the last being a new item in the census; the value of implements and machinery; a classification by tenure of the value of land and buildings, and also of the value of implements and machinery; the number of different kinds of farm animals with some distinctions by age and sex; the number of chickens, production and sale of chickens and chicken eggs, and the number of baby chicks bought in 1929, which is a new item in the census; the acreage and production of various farm crops, and the acreage and value of different kinds of vegetables. Bulletins of this series had been published for 20 states by June 1, and the Bureau expects to complete the series in the summer of the present year.

The second series of these bulletins includes statistics for various minor field crops such as peanuts, soy beans, cowpeas, root crops for forage, etc.; for forest products—saw logs, firewood, etc.; for fruits and nuts; also for dairy products, and for sheep and wool, goats and mohair, and bees and honey. It gives the value of crops distinguishing cereals, other grains and seeds, hay and forage, vegetables, fruits and nuts, and products of farm gardens; the value of different classes of live stock and of livestock products such as butter, poultry and eggs; details regarding mortgage debt and taxes on farms operated by owners; the value of farm products sold and of supplies

purchased through coöperative marketing; farm expenditures for certain specified purposes including feed, fertilizer, farm labor, and electric current; number of automobiles, motor trucks, and tractors on farms; number of farms reporting telephones, water piped into dwelling and into bath rooms, dwellings lighted by electricity; a classification of farms with reference to kind of road on which located, distinguishing concrete, brick, asphalt, etc.; statistics regarding cash rent paid by tenants distinguishing between tenants related to landlord and those not so related; finally movement of farm population distinguishing the number of persons who moved to farms *from* cities and villages and the number who moved from farms *to* cities and villages. The bulletin of the second series had been published for Delaware on June 1. The Bureau expects to complete the series in the fall of the present year, and then to publish United States Summaries of the statistics contained in the first and second series.

The third series of state bulletins presents data for farms classified by type. This is something new in the census tabulations, although a somewhat similar classification was made in 1900. Each farm is classified with reference to its principal source of income. For instance, when the total value of all grain crops sold, or to be sold, such as corn, wheat, oats, barley, etc., constitutes 40 per cent or more of the total value of all products of the farm, the farm is classified as a cash-grain farm. There are 12 main types of farms distinguished on this basis, including general farm, cash-grain, cotton, crop specialty, fruit, truck, dairy, etc. The data shown in this bulletin for each type of farm include number of farms, acreage, value of land and farm buildings, value of farm implements and machinery, value of farmers' dwellings; value of crops, livestock and livestock products sold, and "family living" (value of farm products consumed by farmer's family), specified classes of domestic animals, and a classification of farms of each type by size (under 3 acres, 3 to 9 acres, etc.), by total value of products (under $250, $250 to $399, etc.) and by tenure. The bulletin for Delaware had been published on June 1.

The Tabulation of the Census of Agriculture

In the census of population all the facts regarding each individual, with the exception of those relating to unemployment, are punched on a single card; but in the census of agriculture, the schedule for which includes about 350 items, it is necessary to use 21 different card forms. Many of these are used serially—that is, the same form of card is used separately for a number of different items. For example, the form of card used for tabulating the wheat crop is used also for the hay crop and other principal crops.

The number of cards punched will total approximately 200,000,000. On every card there is punched the color and tenure of the farm operator, the size of the farm within specified limits, the type of farm, and the value of products within specified limits, so that any of the cards may be sorted by color and tenure of operator, by size, by type, and by the value of products before they run through the tabulating machines. The punching of the cards is nearly completed.

The scheme of runs for the various cards would be too detailed for publication in these notes and probably would not be of much interest to readers of this JOURNAL.

J. A H.

MISCELLANEOUS NOTES

The Chicago Chapter.—On February 10, 1931, Dr. William F. Ogburn, President of the American Statistical Association, addressed a dinner meeting of the Chicago Chapter on "The Work of President Hoover's Research Committee on Social Trends." Dr. Ogburn, of the Committee, outlined the purpose, organization and the scope of the work being undertaken by the Committee in the study of causes and effects of social changes. He also spoke briefly of the preliminary findings under a number of the sections of the study, principally those dealing with changes in communication, changes in education, changes in rural life, and changes in religious attitudes and beliefs. Dr. Ogburn said that it was planned to have the final report, which would be contained in three volumes, completed for publication in 1932.

At a dinner meeting on March 10, 1931, Dr. Edwin G. Nourse, Director of the Institute of Economics of the Brookings Institution, addressed the Chicago Chapter on "The Farm Board Experiment." In conjunction with Dr. Nourse's talk, Mr. Marlin H. Poindexter, Vice-President of the Collins Farm Company, Cedar Rapids, Iowa, described the scientific methods employed by his firm in managing 120 farms, over 27,000 acres, in Iowa on a profitable "chain" basis. His talk was illustrated with moving pictures.

Dr. Nourse, in his talk, pointed out that contrary to the common impression only about one-half of the Federal Farm Board's funds have been used in stabilization activities. The remainder has been employed in furthering coöperative activities and in the education of farmers in the control of production. It was his opinion that the actual losses of the Farm Board will be less than $75,000,000. Viewing the stabilization experiment as an emergency measure, this is not an exorbitant cost for the relief of a major industry. The impression of many people that stabilization has been the principal activity of the Board is erroneous and it was Dr. Nourse's view that the Board would not repeat the experiment in that field. Dr. Nourse further stated that relief of agriculture must eventually be dependent upon coöperation within the industry in such things as production control and the use of more scientific methods to reduce costs.

Mr. Poindexter's discussion of methods used by his company was an illustration of the success obtained through the application of industrial methods to grain farming. He pointed out that his company, through the efficient utilization of land and with the aid of machine methods and scientific treatment of the soil and seeds, has succeeded in reducing costs per acre and increasing production at the same time. As a result, it has been able to show reasonable profits at present prices.

The Detroit Area.—The principal speaker at the March 19 meeting of Detroit members of the American Statistical Association was Professor O. W. Blackett. Professor Blackett has just returned from sabbatical leave. During his leave of absence he visited a number of cities in the United States with the idea of determining just how statistics were being utilized in various business establishments. One of the points

which Professor Blackett made was that wherever he went he was most cordially received and that no hesitancy to impart the particular findings of statistical departments was manifest anywhere. Other points which impressed themselves on Professor Blackett during these visits were: (1) that there was a very small demand for statisticians; (2) that where statistical departments had found a place in a business set-up it was invariably because a capable man had started the department. The infrequency of abuses in the use of statistical method was another thing which was evident. Professor Blackett thinks that only the really capable students should be encouraged to specialize in statistics with a view of later becoming statisticians in business if the field is to retain its prestige.

Following the main discussion Professor Blackett responded to a number of questions from members. A point which came out in these questions was that there is no organization which publishes a current bibliography of statistical research work done or being done. Mr. Pryor, one of the members present suggested, and the assembly voted, that the American Statistical Association be approached in the matter of undertaking such a study.

A round table discussion of business as seen through the eyes of the individual members present followed.

The Pittsburgh Area.—A meeting of the Pittsburgh branch of the Association was held on January 22 at the Keystone Athletic Club. There were thirty present for this third meeting to discuss the forecast of business for the Pittsburgh district.

At a meeting on February 26 there was further discussion of probable trends of business in the Pittsburgh area. At this meeting Mr. A. W. Lohr of the Westinghouse Electric and Manufacturing Company was elected Temporary Secretary-Treasurer for this district.

The San Francisco Chapter.—At the eighteenth dinner meeting of the San Francisco Chapter, January 30, 1931, Dr. H. R. Tolley of the Giannini Foundation at the University of California spoke on "Present Trends in California Agriculture." Employing post-war statistics of area and production of the principal agricultural products of California, Dr. Tolley gave a comprehensive survey of trends in the recent past and commented upon the outlook for the near future. Data bearing upon farm land values and farm indebtedness were analyzed in addition to the statistics of crops and live stock. The address represented a summary of recent investigations by the staff of the Giannini Foundation.

At the nineteenth dinner meeting, February 26, 1931, Mr. Arthur B. Griffith of Loomis, Sayles, and Company spoke on the topic, "Industrial Growth and Business Activity as Reflected in Stock Prices." Discussion centered about a chart of the *Annalist* and Axe-Houghton weighted averages of industrial stock prices from 1883, showing monthly ranges on a logarithmic vertical scale; parallel lines had been drawn cutting the tops and bottoms of the undulations; these lines rose at a rate of about 3.3 per cent per year. The record appeared to fall into four cycles of about 12 years each. Downswings represented in each instance a drop of about 50 per cent in the level of stock prices, though in 1929–1930 the drop was over 56 per cent. Comparisons were made with measures of business activity, with reference both to the timing of fluctuations and their amplitude. In "money panics" the bottoms coincided closely in time; in industrial depressions, the low points in stock prices preceded the low in business activity. Questions were raised as to the effect, in the bull stock market of 1929, of the "loans for others" and of the policy of issuing stock dividends.

In conclusion, the speaker stated that the data could easily lead one to believe that the low in stock prices had been reached in December 1930 and later, but that so many elements, differing from period to period, had affected the statistical record that it would be unwise to lean too heavily upon this conclusion.

In discussion, Mr. Adalbert Wolff of Pierce, Fair, and Company brought forward some reasons for questioning the notion that peaks or troughs in stock prices can ever be recognized on evidence afforded by historical time series, and pointed out that other indices might not show fluctuations like those of the Axe-Houghton and *Annalist* indices.

At the twentieth dinner meeting, March 26, 1931, Professor John B. Canning (Department of Economics, Stanford University) addressed the chapter on the topic, "The Meanings of Trends and of Deviations from Trends," with reference particularly to time series in economics.

Neither among the textbook writers nor among the statistical economists does one find substantial agreement as to the content of the concept, "trend." No precise meaning seems even to command a preponderance of followers. To rid the idea of ambiguity two general notions must be distinguished; first, the essential nature of the phenomena appropriately to be dealt with by means of trend analysis; and second, some general definition of the family of functions by means of which trends may be measured. Neither of these general notions has received adequate consideration.

In all trend studies that rise above the level of mere numerical operations one finds the following conditions fulfilled: (1) the events treated have a common, variable character; (2) the events are treated as identical phenomena except with respect to differences in time of occurrence and differences in the values of the common variable; (3) a plurality of non-simultaneous events is considered; (4) measures of the common variable have been made; (5) the measures have been arranged in a sequence corresponding to the temporal order of occurrence of the events; and (6) some defined, representative measure of difference between successive terms in the sequence has been computed.

The fulfillment of these six conditions is sufficient if the trend is a mere expository device by means of which the general sweep of a series of values is to be suggested. But if the trend is to be employed as an instrument of statistical induction additional conditions must be satisfied.

Positive attempts to extend the statement of these conditions have hardly got beyond the stage of suggesting that, to have real meaning, a particular trend must be thought of as a resultant of component factors each of which changes uniformly from year to year, that is, each such factor has a trend of its own. But this throws more light on the nature of deviations from a calculated trend, or on error in relation to trend calculation, than it does upon the general notion of trends.

Negative attempts to extend these conditions, or attempts to free a particular trend measure from particular objections, have taken two chief forms: first, curtailment of series the earlier and later terms of which are clearly non-homogeneous; and second, omitting one or more observations within a series that are seen to have been influenced greatly by some unique episode.

Many economists seem reluctant to curtail a series; they seem to feel that, without a long series, the "law" obeyed by the series cannot be detected or abstracted. The existence of such "laws" seems to have been assumed, but not demonstrated. If a series becomes progressively less homogeneous with increase in time-span, more reliable inductions can be drawn from short series than from long ones—particularly if the induction takes the form of interpolation or of short-range forecasting.

The omitting of one or more observations by reason of episodic influences would command more respect if the omissions did not so often consist of observations that stray widely from the calculated trend. Concurrent episodes of opposed influence must often give rise to observations lying on or near a calculated trend; but if the separate effects of such episodes cannot be measured there is as good reason for rejecting the resultant observation as there is in the case of the "strays" commonly excluded.

The question of what measures of deviation are most significant is hardly opened. With respect to the trend, a value is often called "abnormally large" without considering, apparently, whether it would not be better to consider it "abnormally early." Criteria of goodness of fit cannot well be applied until a decision is made as to whether deviations parallel to the y axis, or parallel to the x axis, or perpendicular to the trend, are most significant.

Activities in Philadelphia.—During the past year members of the American Statistical Association at the University of Pennsylvania have held a number of informal dinner meetings for the discussion of statistical problems. At the most recent of these, invitations were extended to all members of the Statistical Association in Philadelphia and vicinity and to students of graduate courses at the University. About thirty persons were present. Dr. Simon S. Kuznets, Associate Professor of Statistics at the University, presented a paper on the work of the National Bureau of Economic Research, of whose staff he is likewise a member. Mr. Hiram S. Davis, of the Bureau of Industrial Research of the Wharton School of Finance and Commerce is secretary of the group, which plans for the present to hold about four meetings during each academic year.

Committee on Governmental Labor Statistics.—The sub-committee on wages of the Committee on Governmental Labor Statistics, whose work is to set up a plan for governmental collection of statistics of wages comparable with the main Committee's program for employment statistics, met in the offices of the Social Science Research Council on March 9. The members present were Messrs. Meredith B. Givens, D. D. Lescohier, E. B. Patton (secretary), Roswell F. Phelps (chairman), Bryce M. Stewart and Miss Mary van Kleeck. The committee had before it the findings of a preliminary historical survey of the work of the federal and state governments and of the Dominion and provincial governments of Canada in the collection of wages data. The survey showed that in the main, four procedures had been followed—questionnaire to employers, questionnaire to unions, collection by inspectors and information on compensation case records.

The discussion developed the thought that much of the material heretofore presented did not adequately describe actual wage conditions, that the union rates published by various authorities represented minimum standards and that frequently the wage rates actually in force were considerably higher. The committee recorded the view that average daily and weekly earnings, indicated in some wage investigations, had little significance in that they frequently covered workers in many different occupations. It was accepted as a principle for the future work of the committee that it should be directed toward the securing of actual wage rates earned on the job and to their presentation by standardized occupations.

The committee is instituting further research to bring this survey of the wages data of the different governments up to date.

The Committee on Governmental Labor Statistics was recently in communication with the United States Bureau of Public Roads with reference to the securing of

employment data on road construction. The committee was interested to discover that the Bureau of Public Roads has initiated a plan for the collection of these data by which every month the Bureau sends to the road authority in each state a form on which the state officials are requested to list all active projects. When a state has compiled its list, it is forwarded by the state official to the District Engineer of the Bureau of Public Roads as soon after the fifteenth of the month as possible. Two forms are then completed: (1) monthly report of men employed on highway construction and (2) monthly report of men employed on highway maintenance. The forms call for considerable detail. Some states are coöperating by completing the entire report for their state and forwarding it directly to the Bureau in Washington.

United States Bureau of Labor Statistics.—In connection with its regular work on volume of employment, the Bureau has recently begun an extensive study, by cities, of volume of employment and pay rolls in building construction. It is planned eventually to cover a large number of the principal cities of the United States. Data for Washington, Providence, and St. Louis, for the months of December 1930 and January 1931 were presented in the March 1931 *Monthly Labor Review*. The figures for February will include two additional cities—Atlanta and Dallas.

Beginning with the data for January 1931, the method of presenting labor-turnover figures was changed. Instead of using the median of the individual company rates as a form of average, the weighted arithmetic mean is now being used. The Bureau considers this a much better method of presenting the turnover figures. Details regarding the change were published in the *Labor Review* for March 1931.

There has been an increase in the number of cities for which building permit data are obtained, reports being received in January and February 1931 for 342 cities having a population of 25,000 or over according to the 1930 Census. The largest number for which monthly reports on building permits had been received previously was 298.

Data on wages and hours in the iron and steel industry and in the manufacture of silk and rayon fabrics are now being gathered. Field work on the survey of wages and hours in the bituminous coal industry is nearly completed. Bulletins on wages and hours in the sugar-refining industry and in the boot and shoe industry are being edited for the printer, and similar reports are in course of preparation for the lumber, men's clothing, and hosiery and underwear industries. Summary data for the lumber, men's clothing, boot and shoe, and sugar-refining industries have been published in the *Labor Review*.

A study of old-age pension systems in foreign countries is being made, based on published official documents and on reports from the United States consuls in the various countries.

The proceedings at the convention of the International Association of Industrial Accident Boards and Commissions at Wilmington, Delaware, in September 1930, and at the meetings of the International Association of Public Employment Services, held, respectively, at Philadelphia in September 1929, and at Toronto, Canada, in September 1930, are in galley proof.

Bulletins analyzing the labor legislation enacted in various states in 1930, and the important court decisions relating to labor during the years 1929 and 1930, are in course of preparation. A brief summary of the labor legislation of 1930 was published in the March 1931 *Labor Review*.

Women's Bureau of the United States Department of Labor.—Because of the profound importance of the subject of women's wages, from the point of view of business as

well as that of society, and the increasing demand for comprehensive figures whose genuineness is above question, the Women's Bureau has compiled material on the earnings of 100,967 white and 6,120 Negro women working in 1,472 plants in 13 states in which surveys were made by the Bureau during the time from early in 1920 to early in 1925. The report is entitled *Women's Wages in 13 States* and constitutes Bulletin 85 of the Bureau. It was written by Mary Elizabeth Pidgeon, in charge of the division of research.

In each case a representative group in the industries employing the greatest numbers of women was included. In each establishment covered, the actual receipts of all women employed were taken for a week that fell within the same current month or season for all the firms studied in any one state. Every effort was made to insure that the week selected should be industrially normal and representative, should contain no holiday, and should present no unusual circumstances affecting earnings. Despite the magnitude of the task, records were, in all but a few cases, copied directly from pay rolls by agents of the Women's Bureau, in order that accurate and uniform material might be secured within each state.

These pay-roll figures have been assembled, analyzed, and correlated with industry, hours, and other industrial factors, and with age, nativity, and experience of the employees. The findings are shown pictorially in a series of charts. As the surveys were made over a period of five years, in one section of the report the earnings figures are converted to a 1928 basis, by the use of the cost-of-living index of the Bureau of Labor Statistics, for closer comparability.

In general, the findings in the study may be said to signify that the earnings of women in manufacturing—and especially in certain of the important industries that employ many women—are very irregular; that women are subject to much undertime work and their earnings often fall below their rates to a very considerable degree.

Bureau of Research of the New York State Department of Social Welfare.—As a result of a survey made in November 1930, the New York State Department of Social Welfare established on December 1, 1930, a Bureau of Research with Dr. David M. Schneider as its Director. The entire statistical work of the Department is, as a result of a reorganization, centralized in the Bureau of Research.

Monthly reporting systems have since been established for the county, city and town homes (former almshouses), and also for the public agencies administering home (outdoor) relief. On April 15, 1931, monthly reporting systems embracing child-caring institutions and also placing-out agencies will be inaugurated. Plans are being perfected to start a monthly reporting system for the private homes for the aged. This will be changed to a quarterly system as soon as the majority of the existing 180 private homes for the aged will join the Department in this undertaking.

In connection with the Old Age Security operations, monthly statistical data are being compiled. In addition to this, more than 50,000 social histories on applicants for Old Age Relief are being coded, and the valuable sociological data contained in these histories will be tabulated and analyzed by July 1.

In addition to the activities outlined above, the Bureau of Research is revising the record systems for the different organizations and institutions reporting to the Department. The Bureau is also contemplating, as soon as feasible, to send out in the field a full time statistician, who will instruct the agencies reporting to the Department on proper record keeping and statistical procedure.

Developments in the Department of Labor and Industry of the Commonwealth of Pennsylvania.—The Department is centering its industrial accident prevention

activities for the year 1931 around a concerted safety effort to be made during the months of March and October. An examination of Pennsylvania's accident experience over a number of years has disclosed that accidents in industry usually reach their peaks during the months of March and October, and for that reason these two months were selected as appropriate periods for an extended effort for safety in industry. The slogan adopted for the accident prevention campaign is, "Knock Off the Peaks."

Statistical Activities of the Russell Sage Foundation.—Since early in 1929 the Department of Statistics of the Russell Sage Foundation has been collecting and publishing current monthly statistics of outdoor relief in the principal cities of the United States and Canada. These data have become of much interest and importance in connection with the formulation of local emergency relief policies during the past winter. The January 1931, *Relief Bulletin* of the department showed relief expenditures of $8,818,000 granted to 376,000 families in their own homes by 345 public and private relief agencies in 80 cities of over 100,000 population, of which four are Canadian. More than three-fourths of the amount was administered by public relief departments, and additional public funds were disbursed by private relief agencies in several cities. Mothers' aid grants represented 17 per cent of the total amount and old age grants, 5 per cent. The aggregate relief granted by 330 agencies increased 113 per cent from January 1930, to January 1931.

American Committee of the International Union for the Scientific Study of Population Problems.—There has recently been organized an American Committee of the International Union for the Scientific Study of Population Problems. Its chairman is Dr. Louis I. Dublin, Statistician of the Metropolitan Life Insurance Company; Dr. C. E. McGuire is vice-chairman and Dr. Alfred J. Lotka, secretary-treasurer. This is the first working committee organized in the United States to coöperate with the International Union. Its purpose is the study of population problems in the United States, with particular emphasis on those problems having an international bearing.

Annual Meeting of the Public Health Association.—The American Public Health Association announces its Sixtieth Annual Meeting, September 14–17, in Montreal, Quebec, with the Windsor Hotel as headquarters.

The Association has not held a meeting in Canada since 1908, and public health workers from the Dominion and from the United States are invited to take advantage of this opportunity for closer contact. The program is being planned with the progress and needs of both countries in mind. Such subjects as toxoid immunization; rural sanitation, particularly the organization of a practical program for county health units; health education for a large city, for a small city and for a rural community; camp and resort sanitation, including fungus skin infections, particularly those transmitted in swimming pools, and general sanitation of auto camps, have been considered so important by the Program Committee that special sessions will be devoted to them.

Each Section of the Association will arrange individual programs, covering public health administration, laboratory research, vital statistics, public health engineering, food, drugs and nutrition, child hygiene, public health nursing, health education, epidemiology and industrial hygiene.

Meetings of four other organizations—American Association of School Physicians, Conference of State Sanitary Engineers, International Society of Medical Officers of Health and the International Association of Dairy and Milk Inspectors—will take

place during or immediately preceding the sessions of the American Public Health Association in Montreal.

For further information address the American Public Health Association, 450 Seventh Avenue, New York City.

Special Research Fellowships.—The National Tuberculosis Association announces a limited number of fellowships in social research as related to tuberculosis, open to graduate students who have had special training in statistics, social science or public health. Preference will be given to candidates who are interested in pursuing research in public health after the completion of this fellowship.

Researches on topics selected by the National Tuberculosis Association will be conducted in collaboration with colleges and universities, and each study will be under qualified academic leadership.

Academic credit may be allowed for this research according to arrangement with the individual universities under whose supervision they are undertaken.

Each fellow will be required to submit a written report at the completion of his fellowship grant and the text of that report shall remain the property of the National Tuberculosis Association. Candidates will be considered not alone on academic standing, but on experience and general fitness for research work.

The fellowship grants will date from the beginning of the academic year in the fall of 1931. They are for a twelve-month period and the fellowship grant amounts to $1,500 for that period with a month's leave for vacation.

Interested candidates should write to Jessamine S. Whitney, Statistician, National Tuberculosis Association, 370 Seventh Avenue, New York City, for further information.

Eugenics Research Survey.—A critical survey of data relevant to eugenics research is now being prepared under the auspices of the Eugenics Research Association. Mr. Frederick Osborn, Treasurer of the Association, is chairman of the committee in charge of this project. The publication date is tentatively set as January to March, 1932.

In the Eugenics Research Survey special attention will be given to the data bearing upon the operation of social factors affecting the differential natural increase of population groups under different social conditions (demogenics). Dr. Frank Lorimer is in charge of this section of the survey.

The Association will be grateful for the receipt of notices of new research, completed or in process, having possible bearing upon problems of eugenics research, especially in its social aspects. Communications may be addressed to Eugenics Research Survey, American Museum of Natural History, West 77 Street, New York City.

The Brookings Institution.—Mr. Felix Morley, formerly of the *Baltimore Sun* and recently Director of the Geneva office of the League of Nations Association of the United States, has joined the staff of the Brookings Institution. Mr. Morley's study of *The Structure and Functioning of the League of Nations* will be published by the Institution in the near future.

Mr. Lloyd Milton Short, on sabbatical leave from the University of Missouri, is with the Institute for Government Research in connection with studies on agricultural economics and national administration. Mr. Laurence F. Schmeckebier has made for the National Commission on Law Observance and Enforcement, a study of the cost of administration of criminal justice in the District of Columbia. This is one of a series

of uniform studies being made in a number of cities by various collaborators working with the Commission. Mr. Schmeckebier is now making for the Commission a study of the cost of administration of criminal justice by the federal government. Mr. Lewis Meriam is serving on the President's Committee for Employment.

Two reports of the Institute for Government Research on the state government and the county government of North Carolina have been published by the state.

The permanent home of the Institution, located at 722 Jackson Place, on Lafayette Square, was dedicated with appropriate exercises on May 15. In addition to offices and library, the new building provides residential, dining room, recreational, and auditorium facilities. The Institution is now, therefore, equipped to extend its function as a center for research fellows and visiting scholars.

Northwestern University Research Seminar in Paris.—Northwestern University is offering a Seminar in Social Science Research in Paris, from June 1 to July 31, to meet the needs which have arisen with the increase in American research abroad. The value of this research lies not only in the additional data it brings to the student, but also in the new ideas, hypotheses and interpretations which lead the student to question his accustomed point of view. Thus the primary purpose of the Seminar is to lay the foundations for a more intensified international coöperation in the social sciences. Professor William Jaffé will be the Director of the Seminar.

Personal Notes

Dr. R. A. Fisher, chief statistician of the Rothamsted Agricultural Experimental Station, Harpenden, England, will spend eight weeks in the United States during the coming summer. He will be in residence on the staff at the Iowa State College during the first half of the Summer Session, June 16 to July 24, and later will visit a number of colleges, universities and experiment stations throughout the country.

For the past fifteen years Dr. Fisher has been one of the most active among the well-known and progressive group of British statisticans. He has made numerous fundamental and important contributions to the theory of statistics and the mathematics of biological experimentation. He has been successful in solving such problems as the distribution of the various correlation coefficients, including multiple, partial and intraclass. Dr. Fisher's most practical results from these theoretical considerations have included the method of analysis of variance, the general use of degrees of freedom, and the wide extension of the exact treatment of small samples.

His most recent contributions have been in the development of a theory of efficient statistics in estimation, and a more adequate treatment of the mathematics of genetics.

In connection with Dr. Fisher's visit the Iowa State College announces a special group of Summer Session courses covering the theory of statistics and their practical application in biology, economics, psychology, genetics, farm crops, and animal husbandry.

Dr. Egon S. Pearson of the Biometric Laboratory, University of London, is visiting this country this summer, having arrived the latter part of April. He will visit several institutions prior to conducting lectures on Mathematical Statistics during the Summer Session, from June 8 to July 16, at the University of Iowa. Dr. Pearson is well known in this country for his significant contributions to the theory of statistics.

Professor Stuart A. Rice of the University of Pennsylvania has been appointed chairman of the Social Statistics Committee of the Social Science Research Council,

succeeding Shelby M. Harrison. The committee is in process of reorganization. Professor Rice has been succeeded as chairman of the Social Statistics Committee of the American Statistical Association by Dr. Neva Deardorff, Director of Research of the Welfare Council of New York City.

Friends of Donald R. Belcher, Vice-President of the American Statistical Association, will be glad to learn that, after an illness of some six months, he has returned to his work with the American Telephone and Telegraph Company, where he is Assistant Chief Statistician.

Mr. Wendell E. Thorne, formerly Assistant Economist in the Economics Division of the International Telephone and Telegraph Company, has joined the staff of the Division of Research and Statistics of the Federal Reserve Board.

Clytus A. Freeman was recently appointed Chief of the Division of Statistics and Research, Bureau of Customs. The creation of this division of the Customs Service, while it will in no wise duplicate the work of the Tariff Commission or of the Division of Statistics of the Department of Commerce in the compilation of statistics of imports, exports, etc., should meet a very real need by supplying additional information regarding the Customs Service and the revenues derived therefrom which have not heretofore been available. Mr. Freeman for the past several years has been in charge of the monthly statistical inquiries conducted by the Bureau of the Census.

Henrietta X. Kiernan

In the passing of Miss Henrietta X. Kiernan of Washington, D. C., on December 11, 1930, the government sustained the loss of one of its outstanding statistical experts, who had done effective work for more than thirty years in the Bureau of the Census.

Miss Kiernan was a member of the American Statistical Association since 1919. In her official capacity she was associated for many years with the late Messrs. William C. Hunt and W. S. Rossiter, both well-known statisticians. She was well equipped in statistics, having qualified in special subjects in connection with her work at Brown University, Columbia University and elsewhere.

Her sudden and untimely death in her office came as a shock to her many friends and associates.

ADDITIONAL COMMITTEE APPOINTMENTS

Committee to Coöperate with the Social Science Research Council
H. T. Warshow
Louis I. Dublin

Representative of the American Statistical Association on the Council of the American Association for the Advancement of Science
Edmund E. Day

Representative of the American Statistical Association on the National Conference on Nomenclature of Disease
George H. Van Buren

Committee on a Dictionary of Statistical Terms and Symbols
Edwin W. Kopf, *Chairman*
Henry L. Rietz, Mathematical Statistics
Sewall Wright, Biological Statistics

Karl J. Holzinger, Educational Statistics
Henry E. Garrett, Psychological Statistics
Walter F. Willcox, Population
Donald R. Belcher, Business and Economics

Committee on Census Enumeration Areas
Howard W. Green, *Chairman*
(Additional members to be appointed later)

The name of the *Social Statistics Committee* has been changed to the *Committee on Dependency and Child Welfare Statistics.*

MEMBERS ADDED SINCE MARCH, 1931

Aldrich, Harold L., Business Research Department, Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa.
Altman, Alvin, R. H. Macy and Company, 34 Street and Broadway, New York City
Baer, Willis N., Graduate Student, Columbia University, New York City
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Baten, Dr. William D., University of Michigan, Ann Arbor, Mich.
Been, Richard, Crop and Livestock Estimating, Bureau of Agricultural Economics, United States Department of Agriculture, Washington, D. C.
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Brogan, Dr. Albert P., Faculty Exchange, University of Chicago, Chicago, Ill.
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Buller, B. Frank, Hamilton Watch Company, Lancaster, Pa.
Bullian, E. Don, Student, New York University, New York City
Byer, Herman B., Bureau of Labor Statistics, Washington, D. C.
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Hardesty, James D., Statistical work and cost accounting, Ohio Marble Company, Piqua, Ohio
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Secrist, Horace A., Head of Market Research Department, Lewis Manufacturing Company, Walpole, Mass.
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REVIEWS

Population Problems, by Warren S. Thompson. New York: McGraw-Hill Book Company. 1930. 462 pp.

Professor Warren S. Thompson is probably the best informed of the students of population problems in the United States. For a number of years, he has been the Director of the Scripps Foundation for Research in Population Problems and with his associate, Dr. Whelpton, has made significant contributions to our knowledge in this field. He has enjoyed unrivalled opportunities to observe conditions at first hand in various parts of the world and to collect information in many of the danger spots of the world's population. An excellent background of training, observation and years of thoughtful study are all reflected in the volume before us. It will prove indispensable to those who work in the field of demography and it will be many a year before it is superseded. It does credit to American scholarship.

This book is a large one. There are twenty-five chapters and a fairly complete bibliography. The first four chapters are general and historical in character and give the background for the later discussion of current population problems. Emphasis is placed on the contribution of Malthus and the influence of his thinking on the writers of the last century. Chapter five is an excellent discussion of the structure of population. Chapters six to eight discuss the birth rate and the factors in its current tendency to decline. Chapter nine is on the death rate. Chapters eleven and twelve discuss the factors in population growth. Chapter thirteen is concerned with the future growth of population in the United States. Chapters fourteen and fifteen consider agriculture, industry and commerce in relation to population growth. Chapters sixteen, seventeen, eighteen and nineteen consider the city as a population center and its influence on the life of the people. Chapters twenty and twenty-one consider the eugenic aspects of the population. Chapters twenty-two and twenty-three consider the relation of population to international politics. Chapters twenty-four and twenty-five close the book with a consideration of the factors in the control of population growth and discuss the idea of an optimum population.

The subject-headings of these chapters clearly indicate the inclusive scope of the volume. Professor Thompson has combed the literature and has assiduously organized his information on population problems the world over. The book will, therefore, be extremely useful to the general reader who may wish to find a thoughtful discussion of the main topics in this field. To the special student who will take the necessary time to work his way through the volume, it will give a comprehensive grasp of an important field of knowledge which at this time is particularly alive with human interest. The volume does more than present the basic facts of population. It ties them up closely to the everyday life of people, to the current economic forces and in particular to the political relations of nations. It should help to develop sound international thinking and to simplify

the solution of many problems which worry Geneva and the chancelleries of the world. Dr. Thompson is a realist. He knows how troublesome relations between countries often arise because of population pressure, through the lack of natural resources and because of the arrogance of so-called superior races to those less fortunate. In all of these matters, he shows a generosity of feeling and a broad point of view. The book should have a marked influence in spreading a liberal internationalism.

It may seem ungracious to call attention, in a book as useful as this one, to some features which one might wish to see changed in a future edition. There is some discontinuity of treatment in various places in the text. For example, one wonders why Professor Thompson introduced the discussion of the Negro in the United States, as chapter ten. It stands in the way of the following chapters on population growth. This and a few similarly misplaced chapters suggest that the book would have been better if it had been somewhat less discursive. But this is a defect of a virtue rather than of a vice. There is also some difficulty resulting from the author's desire to play up the United States and at the same time maintain a broad international viewpoint. It would have been better, perhaps, to have maintained the international point of view throughout and to have discussed the United States briefly as illustrative of general principles. One also gets the impression, in a few chapters, of rather hasty writing although nowhere is the sense obscure or the reading difficult. It is, however, just as well to have the volume at this time and to look for more polish and careful rewriting in a later edition.

LOUIS I. DUBLIN

Religious Bodies: 1926. Bureau of the Census. Washington, D. C.: United States Government Printing Office. 1930. 2 vols.

This decennial report follows those of 1906 and 1916 with respect to the source of information, namely the local churches. These are also the basic units of the enumeration. They are defined as local groups with a religious purpose and a distinctive membership, whether they call themselves churches, chapels, congregations, meetings, societies, stations, or by some other name.

> The statistics here given relate to the year 1926 or to the church record year most nearly conforming to the end of that year. They were obtained from persons connected with the local church organizations, and include, besides the number of live churches with their membership, statistics of church edifices and parsonages, value of property and debt on same, expenditures classified under two heads—local expenses, and benevolences, missions, denominational support, etc.—and Sunday schools conducted by the local churches.

For the first time churches were separated in the tabulation into two groups; urban (in cities or other incorporated places of 2,500 or over), and rural (outside such incorporated places).

In 1916 two schedules were used, one for churches and one for ministers. In 1926 these were combined in one, and the clergy enumerated were counted only if they were pastors (or assistant pastors) of churches. The facts called for on each schedule were: number of members, distributed by sex and age (below and

above 13 years of age); number and value of, and debt on, church edifices; ownership and value of, and debt on, pastors' residences; expenditures for local and non-local purposes; church schools, i.e. Sunday schools, summer vacation Bible schools, week-day religious schools, and parochial schools, including numbers of officers, teachers and scholars; and the name of the pastor, the number of assistant pastors, or the number of other churches served by the pastor, as well as the collegiate and seminary graduation (if any) of the pastor and assistant pastors. Unfortunately, "it was not found practicable, . . . by reason of the pressure of other work, to undertake a detailed tabulation of the data" concerning pastors. The language inquiry of the 1916 census is also not repeated in 1926.

This report is presented in two volumes in octavo size, a smaller format than in 1916. Volume I presents the detailed statistics of all denominations, for the United States, for states, counties, and cities and separate statistics for those denominations composed wholly or in part of Negro organizations. Volume II consists of summaries of the several denominations, their history, doctrine, polity, and statistics.

Certain changes from 1916 appear in the definition of membership in a number of the denominations, in the form of a shift from communicance to baptism as a basis. In addition, the enumeration of Jewish Congregations was so much more complete than at the former censuses that this fact has necessitated the deducting in certain tables of membership figures for this group in each period, in order to maintain a sounder basis for comparison between periods.

A few outstanding facts from the data presented will be of interest.

The total number of denominations listed in this report is 213, but statistics are presented only for 212. Of the total, 155 are grouped in 23 families and 58 are listed as separate denominations. In 1916 the total number was 202 (with reports on 200 of them) and in 1906, 188. Viewed in the mass the religious bodies of the United States show at each census a picture of mergers, separations, proliferations, and new creations. For instance, between 1906 and 1916, 31 new denominations appear, and 17 old ones disappear; and the latest intercensal period witnesses the recording of 29 new names with 17 old ones erased from the record.

As was to be expected, the population growth revealed by the censuses of 1910, 1920, and 1930, is reflected (in part) in the growth of churches. The basic figures comparing 1926 with 1916 are as follows:

	1926	1916
Number of denominations	212	200
Number of local churches	232,154	226,718
Total church membership *	54,576,346	41,926,854
Value of church edifices	3,839,500,610	1,676,600,582
Total church expenditures	817,214,528	328,809,999
Number of churches reporting Sunday schools	184,686	180,183
Number of officers and teachers	2,167,848	1,952,631
Number of scholars	21,038,526	19,935,890

* Due to the changes in the definitions of membership mentioned above, the figures for this item are not comparable.

Concentration of membership continues to characterize the situation. Of the 212 denominations the 19 largest included 88.8 per cent of all the members. Roman Catholics led with 18,605,003; the Jews came second with 4,081,242, closely followed by the Methodist Episcopal Church with 4,080,777. Then followed the Southern Baptists with 3,524,328; the Negro Baptists and the Methodist Episcopal Church, South were next with 3,196,623, and 2,487,694 members respectively. Six other denominations had a million or more members. At the other end of the scale are the Free Will Baptists (Bullockites) whose total membership was 25 persons.

The percentages of members under 13 years of age range from 0.1 to 45.2, with the average at 18.4; churches with an aggregate of 47,000,000 members report that on the whole, there were five women members to each four men; here again there was a wide divergence among the principal denominations, ranging from 100.4 males per 100 females in the Latter Day Saints, to 32.5 for the Church of Christ, Scientist.

The rural-urban classification presented for the first time in this census bears out the general impressions that monographic studies have created. Though the population of the nation is roughly about half urban, the urban half had only 64,290 churches as contrasted with the 167,864 in rural territory. The opposite is true of the number of members—35,126,927 urban and 19,386,734 rural. Too much reliance cannot be put upon this difference, due to the bias introduced by varying definitions of membership, with the possible weighting of the urban group by denominations, like the Roman Catholic, that have a large percentage of members under 13 years of age, and are largely urban.

For the student of the social situation of the Negro in the United States the section on the Negro church will have much of value. Likewise the growing importance of the city makes the segregation of figures for urban and rural churches of great value. It is unfortunate, however, that the data concerning ministers could not have been carried through tabulation and publication. There is little qualitative information about organized religion. The quality of the clergy, even on so narrow a basis of description as that of these schedules, is of first rate importance, though only the barest educational information is afforded by the schedules. It may be a vain hope, but it seems to the reviewer that the Census Bureau could perform a most valuable service by adding this to the debt the student of organized religion already owes to it.

C. E. Gehlke

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Mathematische Statistik der Personenversicherung, by Gustav Rosmanith. (In: *Sammlung Mathematisch-Physikalischen Lehrbücher*, edited by E. Trefftz.) Leipzig and Berlin: B. G. Teubner. 1930. 141 pp.

This text is intended for beginners in the insurance sciences. It will serve actuarial students, of course; but it is aimed also to provide an introduction to one part of the program in insurance mathematics for students of "insurance science" in general. The Continental universities have had since 1895 an in-

tegrated and constantly improving syllabus in the insurance sciences, intended as a foundation for further specialized study in actuarial science, insurance law, insurance medicine and insurance administration. Our American students of insurance and vital statistics may wish to know where discipline in mathematics (and in mathematics applied to statistics) fits into the general insurance training program.[1] This may be seen from the syllabus itself: (1) Mathematics (pure mathematics through the infinitesimal calculus; mathematics of finance, compound interest, annuities, life and other personal contingencies; the calculus of probabilities or of observations and the mathematics of statistics); (2) Insurance history; (3) Insurance law on a foundation of substantive law in general, the law of contracts, agency, negotiable instruments, corporations, bankruptcy; (4) Insurance accounting; (5) Economics of insurance, including banking practice, investments, the legal, economic and mathematical theories of risk, and social insurance; (6) Insurance administration; (7) Insurance medicine; (8) Insurance politics and public relations, including preventive activities of insurance carriers; (9) Insurance statistics, the subject matter and practical methods of internal and external statistics applicable to insurance; (10) General principles, the philosophy and practices of insurance. This conception of rigorous technical training for insurance service has, so far, received no widespread recognition in the United States, except in a vague way through the actuarial societies.

The scope and the exacting requirements of the insurance training program in Europe have led to the abandonment of what are still mistakenly regarded in the United States as typical European university methods. The older, prolix, "kollosal" texts have given way to abridgments such as Rosmanith's book; teaching proceeds in a more amiable and effective manner; the blatant verbosities of the earlier school have been excised, especially in statistics; insurance students are today given what they really need, not what they or the teacher want or like.

Rosmanith supports the newer school by showing within 141 pages most of the mathematical devices which have proven their worth over the past two hundred years in the adjustment or "readying" of crude experience data in life, sickness, accident, disability and pension insurance. He has excluded nothing important; on the other hand he has given clues to historically justified methods, which do not appear in many of the wordy tomes which insurance students have been compelled to study. These methods may be of interest to workers in other fields than insurance.

In his preface the author defines "mathematical statistics" as that collection of mathematical devices used in adjusting, ordering or smoothing crude statistical data of two sorts; (a) decrement-increment, or survival, series having single and multiple decrement-increment elements; (b) "distribution" series of many kinds (time and frequency series, association arrays, co-variation patterns, and the like). The first division of Rosmanith's classification of "mathematical statistics" has been ignored entirely in all texts available to our American students of "Statistics." Rosmanith's first two chapters deal with the construction of mortality, invalidity and pension tables for closed and open groups, and with

[1] See: *Proceedings of the Casualty Actuarial Society*, November 21, 1930, p. 127, for the French program of insurance studies, including the mathematics of statistics.

the calculations which lead to crude decrement-increment series. The "aggregate," "select," "diagonal" and "truncated" types of experience tables are considered. The elements of algebraic probabilities are set forth, including some devices not shown in the texts available to our British and American students.

The five succeeding chapters (fifty small pages!) give the introductory mathematics and the historical material leading to an understanding of the major "analytical" methods of dealing with survival series. These are the methods of de Moivre, Lambert, Gompertz, Makeham, Wittstein, King, Hardy, Gauss, Euler, Tetens, Karup, Lazarus, Quiquet and Dormoy. Rosmanith's development of Wittstein's so-called "mathematical law of human mortality" occupies one page. That, by the way, is all that is really needed! Most "mathematical laws of human mortality" are at best a mixture of fairly good mathematics and very bad theomatonics.

Mechanical, including graphic, methods of smoothing survival series are discussed in the eighteen pages of chapter seven. These are the methods of Woolhouse, Karup, Sprague, Higham, King and Gram.

The closing chapter discusses the relations between the calculus of probabilities and so-called mathematical statistics. Reading of this chapter is recommended to American students who have been wondering what relation there could be between the two disciplines as taught and practiced in recent years in the United States! An insight into the theorems of Bernoulli and Bayes is given (using illustrative material from Czuber) and into the theory of errors on the Crofton-Gauss-Adrain hypotheses. The use which can be made of *a* theory of errors depends entirely upon one's grasp of the underlying hypotheses. The so-called method of least squares is discussed on two occasions, but in no case is it taken so seriously as to exclude other methods of passing lines among points. Distribution series are treated within three pages; the essence of the Lexis-Charlier and Pearson methods is shown. The treatment of co-variation (sometimes called correlation) requires three pages, and here Charlier's definition, principles and methods are followed. Rosmanith is evidently aware of Jørgensen's[1] observation that the theory of so-called correlation has no rigorous or acceptable foundation! These three pages may be compared with a recently published three-hundred page American treatise on the same subject, which tells us no more than Rosmanith has shown in such short compass.

Readers of the text may expect Dr. Rosmanith next to do a similarly satisfactory job in condensing the mathematics of "insurer's risk." A mass of scattered material, ranging from Jan Nicolaus Tetens to Harald Cramèr and Philip Lundberg, and wholly within the field of mathematical statistics, awaits his attention.

The insurance sciences, by and large, require the services of many scholars skilled in abridgment technique. Practitioners in direct and reinsurance are eager for a text on the mathematics of risk prepared by a discriminating compiler such as Dr. Rosmanith has proven himself to be. American students may

[1] N. R. Jørgensen, *Undersøgelser over Frequensflader og Korrelation*, Copenhagen, Arnold Busck, 1916, pp. 1–4.

be interested in a manuscript bibliography on the mathematics of risk recently deposited with the Library of Congress. Some work should be started shortly by econometricians in rationalizing the legal, economic and mathematical theories of risk.

We may suggest that the mathematical statistics of the future, as applied to insurance and reinsurance, will undoubtedly be the mathematics of risk. A tenable technical foundation for insurances, such as unemployment insurance, involving heterogeneous, catastrophic risk cannot be established without further codification and development of risk theory. Some light may be thrown on this eminently practical insurance discipline when Mr. Arne Fisher completes the second volume of his treatise on probabilities. A beginning has been made by the Italians [1] in condensing the mathematics of risk in general; in insurance teaching we need a condensation of the periodical literature on risk from the insurer's point of view. The real worth of such a text will depend upon the compiler's judgment as to what is durable and worth while. American students of statistics and actuarial science will find much stimulating material in the works of Altenburger, Beneduce, Alfred Berger, Burrau, Cantelli, Cournot, Cramèr, Galbrun, Dumas, Gobbi, Gram, Guldberg, Hausdorff, Landry, Lasheras-Sans, Lundberg, Medolaghi, Vidali, Karl Wagner, Lachmund (1864), and Bremiker (1859).

EDWIN W. KOPF

Birth Registration and Birth Statistics in Canada, by Robert R. Kuczynski. Washington: The Brookings Institute. 1930. xii, 219 pp.

Dr. Kuczynski's study of *Birth Registration and Birth Statistics in Canada* is described as a by-product of the series by the same author under the general title, *The Balance of Births and Deaths*. Of the present work nine chapters are devoted to the development of the Canadian scheme while one chapter is occupied with an analysis of the available statistics.

The author begins by outlining the various efforts made toward the unification of Canadian vital statistics from the time of Confederation in 1867 until 1919, when the engagements were completed between the Dominion Bureau of Statistics and the provinces for the establishment of the present system. A chapter is then devoted to the history of registration laws and vital statistics in each province or group of provinces. By this method the essential features of the Canadian system as a whole and the conditions which brought it into being are succinctly presented.

The Province of Quebec, in which clerical registration of baptisms dates back to 1621, affords material for an interesting chapter. A system venerable in its age and sufficiently complete to reveal over a long period what the author terms "the highest fecundity that has ever been observed in any country" is the picture that emerges. In 1878 the Dominion Census Office, with the assistance of the Abbé Tanguay, issued "the most comprehensive historical study of vital statistics

[1] Federico Chessa, *La Teoria Economica del Rischio e della Assicurazione*, Vol. I, Padua, Cedam, 1929.

that has ever been made," a survey of the yearly number of births, deaths and marriages amongst the Roman Catholic population of the Province of Quebec from the year 1608. Since the Census in Quebec dates back to 1666, the computation of birth rates has been possible from the beginning. Although by an Act passed in 1795 the system of registration was extended to Protestant congregations, a satisfactory degree of completeness was never approached for non-Catholic births, and the completeness of Quebec birth statistics up to the entry of the Province into the national system in 1926 has, therefore, depended on the smallness of the non-Catholic population. Under the French Régime the proportion of the latter was negligible, but in 1831 it comprised more than 25 per cent. In 1921 it had sunk to less than 15 per cent.

The story of birth registration in the remaining provinces is on the whole depressing. The earliest Act, in which civil registration merely supplemented clerical, was passed in Nova Scotia in 1761. But for most of the provinces there was never any approach to complete registration. The efforts of the Dominion Government to obtain at various Censuses reliable birth statistics for the preceding twelve months were foredoomed to failure, though the figures for a particular province sometimes exceeded the number of registrations.

More than once the Dominion Government interested itself directly in registration. The first occasion, characterized by the author as "a most remarkable episode in the history of American vital statistics," consisted in the taking over by the Dominion Government, immediately after Confederation, of the vital statistics of Nova Scotia. For a period of some ten years the Deputy Registrars of the province made returns, not to a provincial office, but to a Dominion Statistics Office in Halifax. For a few years, again, a number of city boards of health were subsidized by Dominion funds to secure and return mortuary statistics; the abandonment of this system in 1891 was followed by a Conference which voiced the desirability of coöperation between Federal and Provincial Governments in the collection and compilation of vital statistics. None of these efforts resulted in a satisfactory continuous system until the Conference in 1918; in the words of the author, "birth statistics for English-speaking Canada up to 1920 are utterly inadequate." (p. 204)

For his estimates of the completeness of birth registration under the system of coöperation between the Dominion Bureau of Statistics and the Provincial Registration Offices initiated in 1920, the author is in the main dependent on the trend of birth registrations and the statements of provincial officials. It may be added that such tests as the Dominion Bureau of Statistics has so far been able to apply indicate weaknesses where the author finds them, in the two provinces of Nova Scotia and British Columbia, and a satisfactory condition in New Brunswick, Ontario and the Prairie Provinces and probably in Prince Edward Island. Quebec did not enter the National Registration Area until 1926, and though the improvement in death statistics was at once apparent, and is now confirmed by the figures for four years, births in 1926 showed a sharp falling off (6 per cent) from the number registered for 1925. It may be noted, however, that for the year 1923, under the old system, there was an almost equally sharp decline in the number of births registered, although in this case a marked recov-

ery was made during 1924 and 1925. Moreover, Ontario births for 1926 showed a loss of 3½ per cent from the previous year. A real decline in the number of births has probably taken place in Quebec as in the other provinces, though failure of some collectors to make or forward the new returns during the first months of 1926 may have exaggerated the downward trend for that year.

An interesting and valuable feature of the chapters dealing with provincial laws is the summary given of the differences between such laws and the Model Act. It was not contemplated, of course, that this Act be adopted in all its details; the Federal Order in Council required only "legislation which conforms to the salient features" of the Act. Nevertheless, it is convenient to find listed in detail the divergences from the exact terms of the Act in the legislation of the various provinces. In some cases the divergences have academic rather than practical interest; in others they are important. For example, the time allowed for birth registration in British Columbia is 60 days. This excessive period has been the subject of discussion with the province and will probably be reduced to the 30 days of the Model Act by pending legislation. It is hoped at the same time to reduce to 48 hours the period allowed in the same province for the physician's notice, which in law is 30 days, though in practice requested within 24 hours.

In the analysis of available statistics contained in the last chapter, the author uses the same method as in his *Balance of Births and Deaths* for Western and Northern Europe, confining himself here, however, to the "gross reproduction rate" instead of the "net." He finds this rate for Quebec in 1928 to be 2.21, about the same as in Germany at the beginning of the century, and computes that at one time in the seventeenth century it must have been 5 or 6. The population of Quebec was very small at that time. The author compares the specific birth rates and "gross reproduction rate" of the Prairie Provinces with the remaining provinces of English-speaking Canada to the considerable advantage of the former, the gross reproduction rates for 1926 thus obtained being 1.72 and 1.37. The latter figure approaches the level of Western and Northern Europe. The significance of the comparison is lessened by the fact that since the reproduction rates are calculated on all women of childbearing ages, whether married or not, and since the Prairie Provinces with their large proportion of immigrant population have a more favorable sex-constitution for the marriage of females, these provinces enjoy an advantage which will not exist amongst the offspring of the women whose fertility is now measured.

It is unfortunate that the analysis of Canadian fertility to be included in the volume of *The Balance of Births and Deaths* announced for 1931 will presumably be made before the results of the Census are available. In ten years important changes in sex and age-composition may have taken place, particularly in the western provinces.

No attempt has been made to check systematically the author's many references, but from a somewhat close acquaintance with the present registration laws and a general knowledge of the course of registration history in Canada, the thoroughness and accuracy of Professor Kuczynski's work may be unequivocally endorsed. A few minor lapses only have been noted. On page 24 the author refers to the legislatures of the North West Territories and the Yukon Territory. While the

Yukon Territory has an elective Council of limited powers, the North West Territories have no elected representatives; both are administered by the Dominion Government. On page 213, British Columbia is included under the heading "Maritime Provinces," a term restricted in Canadian usage to Nova Scotia, New Brunswick and Prince Edward Island. On page 50, footnote number 49 states that for 1851–57 the figures quoted by the author for all baptisms in Quebec Province are even lower than the Dominion Office's figures for the Catholic births alone, but this statement is not valid for the years 1851 and 1852. On page 62, concerning the changes in Quebec legislation effected in 1924, we are informed that "children not baptized at all or not baptized by a person authorized to keep registers are no longer registered by the municipal secretary treasurer but by a 'collector of vital statistics.'" The duty of registration was not affected by the new Act, and the new returns are not registrations.

Every student of Canadian vital statistics is under obligation to Dr. Kuczynski for a volume which contains within small compass the results of so much skilled and painstaking research, especially in view of its full documentation and profusion of explanatory footnotes. The presentation of the various registration laws in tabular form adds materially to the value of the book for reference purposes.

R. H. Coats, *Dominion Statistician*

Elements of Practical Statistics, by F. H. Harper. New York: The Macmillan Company. 1930. xix, 324 pp.

This text-book on statistical methods is the result of a thoroughly commendable desire on the part of the author "to remove the mathematics of elementary statistics from the realms of theory and hieroglyphics and to pave the way for a ready grasp and keen appreciation of the more advanced phases of the subject." The attempt at simplicity, however, usually takes the form of an encouraging phrase such as, "This is a very simple matter," and the confining of the explanation of a given technique to working out in tiresome detail a numerical example of the technique described, with inadequate attention to general principles. The student is assumed not to be familiar with algebra, even to the extent of solving a pair of simultaneous linear equations, or even of evaluating the dependent variable in a single equation, for a given value of the independent variable: yet complex curvilinear trends are introduced, as well as multiple and partial correlation. Such a student would find it utterly impossible to follow many of the illustrations given.

The reviewer would raise a question as to the wisdom or practicality of the attempt to "simplify" statistical technique to the extent of relieving the student of the responsibility for mastering the elements of algebraic manipulation. In teaching accounting, we assume a knowledge of arithmetic; in studying the drama or public speaking we require a basic knowledge of English composition: is it fair to a student to lead him to believe that he can master trend fitting, or correlation, or regression, without familiarity with the language—the short-hand —of elementary algebra? Better, have a preliminary review of these matters.

Data for illustrative problems are taken mainly from the field of agricultural economics, yet often absurd hypothetical data are used which poorly represent the problem being considered. No discussion whatever is given of seasonal variation, and the treatment of index numbers (in seven pages) is entirely too brief: the author's statement that other index number problems are beyond the scope of the text is inconsistent with the devotion of 89 pages to correlation and related subjects. The organization is sometimes faulty, as when time series and spatial distributions are considered in the chapter entitled "frequency distributions."

Just as important is the existence of dozens of errors throughout the book, some trivial and others fundamental. The definitions of the following are either incorrect or faulty in some respect: random sample, a variable, normal frequency distribution, skewness, progressive mean, mean deviation (p. 108), correlation, coefficient of alienation, regression. Errors occur in the explanation of the moving average and in the formula for the mode of a frequency distribution. The author repeatedly insists that correlation deals only with cause and effect relationships between the variables being studied! The treatment of multiple correlation is hopelessly confusing: furthermore in one illustration dealing with four variables the *ignoring* of the possible correlation between one of the independent variables and the other two ("This is because that (sic) over a period of years it is reasonable to suppose there will be no causal relation between these two factors") is interpreted as *holding constant* this variable. The meaning of the "path coefficients" is not clear; it is clear only that they do *not* represent, as stated, "the net effect of both November and December price upon acreage." The whole discussion of measuring the error of estimates from correlation problems is utterly fallacious: attention is directed to the difference between the *averages* of actual and estimated values instead of using the average (standard error, usually) of the individual differences. These are typical of literally dozens of mistakes, many of them just as serious, which would require pages to present.

The book is so amateurish and deficient when measured by any and all tests for a text-book that one is led to wonder how it ever came to be published by a reputable house.

WIRTH F. FERGER

University of North Carolina

Statistics in Social Studies, edited by Stuart A. Rice—for the American Statistical Association. Philadelphia: University of Pennsylvania Press. 1930. xii, 222 pp.

This book is exceptional, and welcome, because of the stress laid on those problems of method and implications of procedure which are too often slurred over or ignored. Few numerical results are given and there is comparatively little reference to technical detail of statistical rates and averages; therefore, the papers composing the book should be readable by a public much wider than that which usually lifts the covers of statistical texts.

The twelve papers were obtained by the Committee on Social Statistics of the

American Statistical Association for the meetings in December, 1929, in an endeavor to show what statistical research was then taking place in the social sciences and how problems of scientific method were being dealt with in that research. With the exception of Paper I and of Papers VIII to X on Prohibition, each paper describes research in a different field, but each concerns itself with the fundamental problem of finding something measurable or countable which shall correspond as closely as possible with the phenomenon which is the primary object of interest.

Paper V on Race Relations may be particularly wide in its appeal, not only for its subject, but also because its discussion of scientific fundamentals is easily adaptable to other subject matter. However, each paper is suggestive in its own way, and should help the student or layman to cut his statistical teeth, i.e., to learn to judge of the value of any statistical results which he may encounter. Papers I and IV, "Historico-Statistical Approach to Social Studies" and "Statistical Studies of Dependency," are closely related and should be read together, while II and XI, "Statistical Studies of Marriage and the Family" and "Statistical Studies of Social Attitudes and Public Opinion," have slightly more concern with technical detail than others.

Paper VIII "Prohibition: Statistical Studies of Enforcement and Social Effects" differs from the rest in stressing numerical statistical results, and arguments therefrom, while paying far too little heed to scientific method not only in its discussion but in the thought processes behind its procedure. However, it might well form a useful test of the critical power awakened in the reader by the preceding papers, if he notes in passing the back-slidings from scientific approach implicit in Paper VIII and then compares his criticism with those in Papers IX and X, which are discussions of Paper VIII.

These papers make no pretensions to being more than beginnings in their respective fields, and they tend to show evidence of the necessity they were under of construction for delivery on a given date, but as beginnings they serve a useful purpose.

MARGARET H. HOGG

Russell Sage Foundation

Railroad Purchasing and the Business Cycle, by John E. Partington. Washington, D. C.: The Brookings Institution. 1929. 309 pp.

This book is an attempt to test inductively the thesis that "alternating periods of expansion and depression in the railroad equipment industry have exercised an important influence upon the oscillations of general business in the United States." This task involved efforts in two directions: (1) the gathering of reliable data on the activity of the equipment industry, (2) the fixing of definite dates for the oscillations of general business in this country, to enable precise comparisons between the latter and the cyclical fluctuations in railroad equipment.

The data used to study the volume of activity in the equipment industry were the orders reported in the trade journals, as well as those received by the Baldwin Locomotive Company whose books were made available to the author. Four

types of equipment were covered: rails, locomotive, passenger cars and freight cars. Care had to be exercised in dating the orders as reported in the journals, checking them to avoid duplication, assigning them to the proper railroad and excluding foreign purchases. The data finally obtained yield series of quarterly orders for each of the four types of equipment from 1870 to 1926. Monthly data on locomotives ordered from the Baldwin Locomotive works are also given from 1870 to 1900. The sample thus covered varies in representativeness, being more complete after 1900 than during the preceding years.

The description of fluctuations in the activity of the equipment industry as revealed by these data on orders is supplemented in the discussion by quotations from trade journals on the state and prospects of the trade. To enable precise comparison, the oscillations in general business conditions were studied and turning points assigned at precise months. Also, an index of railway gross earnings (quarterly) was compiled for the same 57 years and the relationship studied between earnings and orders for equipment.

The most important conclusion of the study is the established tendency in railroad buying to precede both revivals and recessions in general business. This lead was more conspicuous in recessions than in revivals and more apparent in the nineteenth century than in the twentieth. As far as the amplitude of fluctuations is concerned, locomotive purchases showed the greatest tendency to fluctuate with changes in business conditions, rail purchases the least. There has been no definite trend in the degree of fluctuations in railway buying accompanying changes in business activity. Finally, in comparison with earnings, railroads have bought most freely when their earnings were large, changes in the volume of orders generally preceding similar changes in gross earnings.

These conclusions point definitely towards a diminished significance of railroad purchases as a factor in the general business cycle. It is rather surprising that there was no decline in the degree of cyclical fluctuations in railroad purchasing. One wonders whether the changing character of the sample may have had something to do with this conclusion.

A great deal of care and labor have been expended in making the inductive test as reliable as possible. But some questions still remain. Can one trust the conclusions for the early years of the series, when the sample represents only a small part of the field? May there not be doubt as to the value of comparing turning points in the series of orders with the turning points established for general business? The concept of a general business cycle is a vague one, when pinned down to definite months as the points of peak and trough. Thus, comparing the author's turning points with the ones established by the National Bureau of Economic Research in its study of business cycles, we note that his list omits the contractions of 1887–1888, 1893–1894, and 1899–1900 noted by the Bureau. Also some of the turning points in the two lists are a few months apart. Might it not have been better to compare the orders with some important single series, so that both the average lag or lead and the tendencies in it could be established not only in comparison with a general index, whose homogeneity through the long period might always be disputed, but also in relation to specific series of general importance, such as pig iron production or bank clearings?

These statements should not be taken to mean that the conclusions drawn by the author are to be seriously doubted. They have been derived in full recognition of the limitations of the data. But one tends to regret that, in view of these limitations, no attempt has been made to supplement the inductive test by an analytic consideration of factors making for interrelationship between railroad buying and business cycles. Tentative as such consideration might have been, it would have made the presentation more interesting, particularly by suffusing the annals derived from trade journals with possible interesting revelations as to the interworking of factors in general business and in the planning of railroad activity.

As it stands, the book is a thoroughgoing attempt to study inductively the fluctuations in railroad purchasing. The relating of these fluctuations to changes in business at large has been carried through in a somewhat restricted fashion. Some important and valid empirical generalizations stand out. They call for amplification and an analytic treatment of the topic.

SIMON KUZNETS

National Bureau of Economic Research

Municipal Expenditures, by Mabel L. Walker. Baltimore: The Johns Hopkins Press. 1930. 198 pp.

This study represents an attempt to discover a pragmatic norm for a rational distribution of municipal expenditures. Starting from the accepted application of the marginal principle to public finance—that the last dollars expended on various governmental functions shall all have an equivalent social utility—Miss Walker is concerned to determine how the marginal utility of municipal services may be measured. In fine, to the old criteria of honesty, economy and efficiency is added a fourth standard of excellence of administration—proportion, balance, symmetry of development.

The measure of the marginal utility of a governmental service proposed is found in the average proportion of the budget devoted to it in a number of different cities—just as markets determine the value of a commodity—no more being claimed than that this affords a rough approximation. To measure the grade of governmental service enjoyed, indexes of rating are developed by means of selected functions measurable quantitatively in terms of results rather than methods or costs. Objective tests of municipal performance are thus secured through twelve roughly weighted criteria, such as number of times a week for street cleaning and garbage collection, mortality rates for health, percentage enrollment for schools, etc. The application of these service indexes to some 160 cities gives us their service ranking, the cities being arranged according to size groups in a series of revealing tables. When city ratings are related to the various factors affecting them, such as size, geographical location, form and cost of government, a number of highly interesting relations emerge; such as a not unexpected one between the cost of government and the service rendered, for generally speaking good government is costly government, and in less degree one between per capita wealth and service rating. Yet the relation between the

three factors of service, wealth and costs is not very strong, cities being found which are low in one of the three respects and high in the other two. Chapters discussing tendencies in budget distribution and limitations, internal and external, on budget making, complete the study.

There is much in this book with which not every student of municipal affairs will agree; one might, for instance, question the criteria of service erected, and the adequacy of some of the statistical data employed. But Miss Walker is herself laudably conscious of the intractability of much of her material and the limitations of her methods, and exercises consistent judgment and common sense in drawing her conclusions. By this careful piece of pioneer work, she places students of municipal government in her debt both for its intrinsic interest and for an essay in the comparative method quantitatively applied to city expenditures.

ARTHUR D. GAYER

National Bureau of Economic Research

Consumption of Meat and Meat Products, with Particular Reference to Price, by John H. Cover. One of a series of studies in the packing industry, under direction of the Bureau of Business and Economic Research of the University of Chicago, and financed by the Local Community Research Committee of the University jointly with the Institute of American Meat Packers. University of Chicago Press. 1930. 81 pp.

This monograph is an interesting application of modern statistical technique. It is largely concerned with statistical relationships; interpretation and causal influences are seemingly reserved for future study. The study centers upon retail prices of five beef cuts, and pork chops, lard, bacon, and ham. Other packing-house products, edible or inedible, are not discussed; nor is any consideration given sheep and calf products. The basic data are the retail price series of the Bureau of Labor Statistics, 1913–1928. Wholesale prices are not considered. In short, the field covered is much narrower than the title suggests.

Retail prices of the stated meat foods, adjusted for the changing price level, have not greatly increased, relative to meat substitutes. Indeed, the deflated or adjusted prices of bacon, lard, and pork chops are less than the 1913 relatives. Much more to the point, however, would be a comparison of retail with wholesale prices of meat foods, and with prices of live animals. Changes in the average dietary have no doubt affected the demand for certain pork products, and for this reason even the rise in absolute prices is worthy of note. Be all this as it may, the study develops a number of suggestive relationships—trends, cyclical and seasonal movements, regional price differences of the various meats, and the like.

Of particular interest are the data respecting the relationships between production, prices, and consumption. Unfortunately, the official data upon production—the only data available—are so unreliable as to impair the validity of the conclusions. To be sure, the statistics for the federally-inspected plants are adequate, but the estimates for at least 40 per cent of the national meat

supply are largely conjectures. There has been no census of retail slaughter since 1909, there are no recent data upon customs slaughter, and the statistics of farm-killed meats are rough in the extreme. To illustrate, the census of manufactures, upon which the official estimates are based, includes something over 1,200 plants. Yet the total number of plants reported to us by five authoritative state sources exceeds the total number reported by the census for the United States; and these five states do not include the most important meat-packing areas. No doubt many of the excluded plants are relatively small, but until the completion of the decennial census of 1929, wherein an attempt was made to supply the gaps in the available data, it will be impossible to determine the margin of error. The same objection holds true for most studies of meat production and consumption—the basic estimates are hardly more than guesses.

L. B. Zapoleon

Food Research Institute, Stanford University

La Prévision du Prix du Coton Américain, by Roger Baury. Paris: Epinal. 1930. 38 pp.

This pamphlet is a correlation study of the price of cotton and the factors affecting it, using data for the years 1919–1930. The author says, "It is difficult to go back of that date because the economic dislocation caused by the War, rendered it practically impossible to study the earlier period." (Page 5.) The monthly price series used was the average of daily closing prices of middling on the New Orleans Exchange. As a first step, these were corrected for purchasing power of the dollar by dividing through by the *Annalist* index number of wholesale prices of farm products. Monsieur Baury then determines the effect on price (corrected for purchasing power) of the yield per acre and the carryover at the end of the crop year. He assumes that there is a hyperbolic relationship between these variables of the form

$$xy=k$$

where x is price, y are the factors, and k is a constant. The y term is a function of yield and carryover and equals $0.107R+0.565CO$, where R is the yield per acre measured in hundredths of a bale, and CO, carryover, is expressed in millions of bales. The equation then becomes

$$x(0.107R+0.565CO)=k.$$

Roger Baury's use of the hyperbolic relationship is ingenious. This equation takes account of the purely internal factors determining the price of cotton. He next considers external factors. "It will now be shown that the left hand side of this equation is connected with the right hand side by a linear regression equation, and that the right hand side k is nothing other than the index of industrial activity." (Page 23.) His choice of an index of industrial activity is curious. It is the old U. S. Bureau of Labor index of employment, corrected for seasonal variation (not the new Federal Reserve Board index adjusted to census totals). "This index reflects exactly both industrial activity and purchasing power of the American wage earners . . ." "It may be considered excellent in spite of the opinion of certain American statisticians." Replacing k

by a linear function of this index, Baury finally gets the equation

$$xy = 1.1208A - 10.19$$

where A is the *Annalist* index of employment corrected for seasonal variation. "We thus obtain a first relation between 3 elements: Corrected price, cotton factors, and industrial activity." (Page 25.) The relationship is represented by a very neat three dimensional diagram on page 26.

In working the correlation, the price was smoothed by a 5 months' moving average. One hundred and twenty-eight observations were thus obtained and the correlation was found to be 0.8193. When seasonal variation is eliminated from the price, and annual means for the crop years are taken, 11 observations result and the coefficient of correlation is found to be

$$R = 0.9945 \pm 0.002.$$

This pamphlet probably fills a gap in French statistical literature by providing an example of applied mathematics in economics, but it certainly adds nothing to the published work of Bradford B. Smith, H. L. Moore, and others. Baury's correlations are largely spurious. The correlation using the annual means has only 11 observations and there are 5 independent variables. No trends were eliminated. It so chances that the general trends of both cotton prices and the U. S. Bureau of Labor employment index have been down since 1919. As is now well known, the U. S. Bureau of Labor employment index has a considerable bias in the downward direction, and is becoming less and less representative of what it is supposed to measure. Its very imperfection makes it perfect for Baury's purpose. (One cannot help wondering whether the real reason for not pushing the investigation back into earlier years, is that this so convenient index was not available prior to 1919.) The correlation between price and yields uncorrected for trend, is likewise spurious, due to the non-elimination of trend. We need not be surprised then, if Baury's equations cease to maintain such a high agreement between predicted and actual prices in the future.

VICTOR VON SZELISKI

Oil Conservation and Fuel Oil Supply. New York: National Industrial Conference Board, Inc. 1930. xiv, 165 pp.

Despite the fears of the conservationists of a decade ago regarding the probable early exhaustion of our petroleum reserves, the American oil industry has for long witnessed an uninterrupted increase in its annual output of crude oil, annual production having increased from 378,000,000 barrels in 1919 to 1,007,000,000 barrels in 1929. Even this tremendous increase in domestic output would have been inadequate to have supplied current gasoline consumption had improvements in refining efficiency not made a barrel of oil go further. Under the old "straight-run" methods of refining with a production of 18 per cent of gasoline from a barrel of crude oil, it would have taken more than twice the amount of crude oil to have produced the 1929 output of gasoline. The introduction of the "cracking" process made possible an average output of 39 per cent of gasoline from every barrel of oil refined in 1929. If the more efficient "cracking" process should be applied to all the oil currently refined, it is estimated that a 60 per cent

recovery of gasoline would be achieved. That this does not occur is a reflection of the fact that oil production has increased so rapidly that it has not been found necessary to extract other than the easy-to-get values from a barrel of crude. The partially refined residues have made available an increasing volume of oil for fuel purposes.

Fuel oil has pushed its way into the expanding markets for fuel materials. It has done so along with gas and water power at the expense of coal. The expansion in the consumption of these former materials accounts in considerable part for the fact that the coal production of the world and particularly that of the United States and Great Britain has remained almost stationary since 1913. This study by the National Industrial Conference Board involves a detailed statistical analysis by regions and industries of the facts in this expansion of oil in the world's fuel markets, with especial attention to those of the United States.

But as fuel oil has come to play a more important rôle in the fuel markets of the world, shifting centers of oil output, stabilized production and more complete refining methods—particularly the hydrogenation process which promises 100 per cent extraction of gasoline from a barrel of crude—threaten a decline, so the authors believe, in the domestic consumption of fuel oil. They expect the price factor to bring a shift from fuel oil to its various competitors as a source of energy and heat.

The concluding part of this volume is devoted to a study of the probable effects of a decline in fuel oil supply. The conclusion reached is that marine and domestic heating will be the most persistent users and that manufacturing and transportation uses will show the most marked decline. The regions likely to be most seriously affected are those remote from centers of large gasoline consumption. More complete refining methods are likely to be accompanied by the transportation of a larger percentage of total crude oil to population centers for refining purposes. The predominance of coastal refining will make a large part of the fuel available for the most essential marine market. The regional and industrial shifts from fuel oil to other fuel materials are expected to be gradual and to be brought about in response to the price factor. They may be expected to ease somewhat the position of the coal industry which has been so precarious during the past decade.

The study is replete with statistical and graphic material and should prove of interest and value to anyone concerned with the market problems of the major fuel industries.

GEORGE WARD STOCKING

University of Texas

Social Work Year Book—1929, Editor, Fred S. Hall; Assistant Editor, Mabel B. Ellis. New York: Russell Sage Foundation. 1930. 600 pp.

The first yearbook on social work is additional evidence that social work as a profession is coming of age. The heterogeneous fields included within the scope of social service have long demanded a directory of information such as is herein contained. It will serve to make the general public better acquainted with the many directions in which social work has spread.

The book is divided into two parts: first, covering 187 topical articles; and second, a directory of 452 national agencies, public and private, operating in the field of social work. Topical articles cover such groups as families or adult individuals, children, the handicapped, miscellaneous classes such as immigrants, the aged, rural social work, etc.; mental hygiene, health, industry, crime and penal conditions, recreation and related activities, community organization, church social work, and other unclassified topics.

Each article is signed and includes, in addition to definitions, the history and present status of the movement, together with developments and events in 1929. There is a complete system of cross-referencing and bibliography.

The treatment of each topic, while brief, gives the reader an excellent understanding of the subject and supplements this by directing him to other sources of information.

The contributors make up a distinguished list of persons who have achieved standing in every phase of social work.

The volume will be of use not only to social workers, but to the students of sociology and those engaged in social research, as well as to many other groups who are operating on the fringe of social work or in professions which are served by it.

The labor involved in putting it out must have been tremendous. The Russell Sage Foundation and Editors are to be congratulated on such a valuable contribution to the literature.

MAURICE TAYLOR

Financing the Consumer, by Evans Clark. New York: Harper and Brothers. 1930. 347 pp.

Mr. Clark's book reviews the rapid development in recent years of the various types of agencies which have arisen for the purpose of advancing credit to the consumer. The growth of such agencies as the personal finance companies, the pawnbrokers, the industrial banks, the personal loan departments of commercial banks and the credit unions is discussed in detail, and figures are given regarding their present position and recent trend. Owing to the difficulties inherent in any inquiry of this sort, many of the quantitative data are admittedly estimates, based upon intelligent guesswork. Enough is presented, however, to indicate the importance of the subject and the necessity of securing more specific information.

There is a painstaking analysis and comparison of the costs, charges and profits of concerns in the small-loan industry. Here again, through no fault of the author, the enormous difficulties in the way of collecting information require caution in the interpretation of the statistical results. Concealed charges of one sort or another, confusions between interest and discount rates, differing rates upon loans of different sizes, the wide dispersion among firms even in the same branch of the business, all serve to complicate the "indexes of cost to borrowers" which Mr. Clark sets up. Such indexes vary from 6 per cent per year in the case of certain credit unions to 480 per cent on loans made by some of the unlicensed lenders. The statistical difficulties are fully recognized by the author,

and form the basis of his plea for the collection of more detailed and accurate information. The rest of the conclusions are concerned mainly with the present chaotic state of public policy toward the small-loan business, and with suggestions for improvement and regulation, including a recommendation for government aid to credit unions. In the appendices are to be found summaries of the loan legislation of the states and texts of uniform laws.

In spite of a natural leaning toward credit unions—Mr. Clark is Director of the Twentieth-Century Fund—the author has managed to maintain a remarkably unbiassed attitude in his discussion of the rival forms of credit agencies. There is evidenced at times, however, a somewhat uncritical attitude toward the movement as a whole that robs the study of part of its objective quality. There is perhaps an overwillingness to assume that the fact of repayment is a *prima facie* indication of the desirability of the extension of loan facilities. But neither criticisms of this nature nor emphasis upon the qualifications that must be attached to the statistical conclusions should be allowed to obscure the fact that this book represents a careful and valuable excursion into a field about which too little is known at the present time.

Douglass V. Brown

Harvard University

Investment, by L. L. B. Angas. London: Macmillan and Company, Ltd. 1930. 184 pp.

Mr. Angas is a retired member of the London Stock Exchange and is an eminently practical observer of stock market phenomena. His book constitutes an exposition of his theories as to how and when common stocks may be bought and sold with a reasonable expectation of profit.

The discussion is more nearly one of speculation, or of trading, than of investment as the term is generally used and understood in this country. This observation remains valid, even though a considerable portion of the material presented deals with "long-term" operations.

Despite certain objections which may be taken to it, the study, considered as a whole, must be judged as one which is decidedly helpful to any student of security market problems. Outstanding among its merits is its conservatism. Despite the fact that the author is for the most part dealing with what we in this country commonly regard as trading operations, his emphasis is consistently on the side of caution. Examples:

"Only buy the market leaders." (That is, avoid what in Wall Street are commonly but expressively referred to as the "cats and dogs.")

"Prefer geometric industries." (That is, industries in which the estimated growth is greater than the average.)

"Always beware of booming trades."

"Insist on earnings of 8 per cent." (Mr. Angas refers to earnings as a percentage of selling price. In America we are accustomed to express earnings as a ratio; earnings of 8 per cent are equivalent to a price earnings ratio of 12½.)

"Beware of companies if their profits have risen continuously for four years."

"Prefer the second year of revival." (That is, buy too late rather than too soon.)

"Follow your pessimism and distrust your optimism."

"One should cut losses before they become too big."

"If paper profits are shown one should not be too greedy."

Following such market advice doubtless would result in the missing of a considerable volume of *possible* profit, but it would probably not send very many stock purchasers to the almshouse.

Other merits of the book are: (a) its comprehensiveness—it touches many, many phases of the stock market problem; (b) its brevity—it is virtually all solid meat, with the minimum of argument and explanation; and (c) its clarity of exposition.

On the other hand, the writer's statements of his conclusions and his hypotheses appeal to this reviewer as much too confident and positive. At best, both speculation and investment are still experimental sciences, if indeed they can claim the dignity of sciences at all. The many thousands of man-hours that have been employed in careful security market research during the past two decades or so have by no means been wasted. Certain conclusions have been developed which may be entertained with a considerable amount of confidence. But outside of these there is a whole host of principles and hypotheses which must still be entertained cautiously, tentatively.

Mr. Angas is not at much pains to differentiate between the secure principles and the tentative ones. He grinds them all out of the same hopper, and democratically offers them for general consumption, without direct warning as to the points of possible fallibility. This is the greatest objection to his report.

LAURENCE H. SLOAN

Principles of Engineering Economy, by Eugene L. Grant. New York: The Ronald Press Company. 1930. xiv, 387 pp.

In response to the criticism that economic aspects of engineering are not sufficiently emphasized in the training for that profession, this book is offered as a text for a new course to supplement conventional courses in general economics and accounting. It is not a text on economics, management, statistics or accounting, but brings the principles and techniques of these and other fields into a consideration of numerous problems involved in estimating the economy of engineering projects. Part I, The Arithmetic of Engineering Economy, considers such items as depreciation, increment and sunk costs (unusual terms suggesting but not synonymous with variable and fixed respectively), load factor and replacement; Part II, Fact Finding in Economy Studies, presents the elements of accounting, statistics and other methods of recording, finding and analyzing data; Part III, Background for Economy Studies, considers financing, forecasting and other "judgment factors." The book is excellently designed for its purpose and should prove serviceable in schools of business administration as well as engineering schools. It is singularly free from mathematical formulas and other abstractions, and is essentially a collection of problems and cases; it

gives primary attention to long-run rather than immediate costs; and, avoiding the temptation to become a compendium of engineering data, emphasizes principles and methods. Because it is concerned with elementary standard practices, its treatment of statistics and other subjects does not call for special comment.

H. S. PERSON

Facts About Filipino Immigration into California, prepared by Louis Bloch. Special Bulletin No. 3, State of California, Department of Industrial Relations, San Francisco. 1930. 76 pp.

Coincident with the restriction of immigration from Europe, the inflow of Filipinos has increased, totalling 31,092 through the ports of San Francisco and Los Angeles in the decade ending in 1929, with the peak in that year. These Filipino immigrants were preponderantly young, male and single. Seventy-nine per cent were between 16 and 30 years of age; 93 per cent were male; and 77 per cent were single. They enter chiefly into hotel, domestic and agricultural occupations, such as asparagus cutting, competing to some extent with Mexican and native white labor. By 1929 considerable anti-Filipino sentiment had developed, culminating in riots in two California communities.

This report is a simple, clear statement of the facts concerning this "wave of Malay immigration," with the numerical data fully set forth and effectively presented in numerous simple charts.

HARRY JEROME

University of Wisconsin

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